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ENERGY

WORK, HEAT AND TRANSFORMATIONS

BY

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PREFACE

The earlier chapters of this work are self-explanatory. The later ones justify some discussion of point of view.

The writer is not a physicist. Educated and trained as an engineer, his call to the teacher's chair led him to arrange his views as to natural principles with greater care than is common with engineers. The ideas promulgated in the following pages are in answer to questions received from bright-minded students a dozen years ago—questions which were sensibly asked, but which "stumped" the teacher for years for an adequate, equally sensible reply.

His efforts at the comprehension of thermodynamic action have led him to trespass, perhaps, upon the domain of the physicist. For the discussion of matters so intricate as molecular dynamics a thorough familiarity with experimental and mathematical physics might seem indispensable. No one can regret more than the writer his lack of this. His apologies for the consequent shortcomings of this little book are profuse in proportion.

Yet, should this situation arouse criticism or doubt, the answer is easy. Why have not the professional physicists long ago done this same thing, that it might have been done far better? The materials, opportunity and demand have long existed. Whatever question may arise as to the significance, or even the definition, of the more recent data of experimental physics, there can be none as to the long established principles of celestial mechanics. There is as little as to the only less venerable data as to thermal processes. For nearly a century there has been virtually no question as to the mechanical nature of heat. Yet these things cannot be accepted by any teacher of thermodynamics without enforcing conclusions substantially different from those now commonly taught. Therefore the writer cannot regard the concepts set forth in this little book as aught else than the indispensable premises for, rather than the remote conclusions from, experimental and mathematical physics.

The writer would never wilfully question the doctrine that accurate data are essential to progress. Scientific concepts cannot

advance without them. But what has been forgotten is that they are a means to an end, not an end in themselves; and that end is the better understanding of nature's ways. Science serves humanity only as it substitutes scientific concepts for superstitious guess-work. The accumulation of unending columns of figures and physical constants, even with religious accuracy, is as futile for the uplift of the race as is the accumulation of vast hoards of dollars, however conscientiously accounted to the last penny. Each *may* be turned to humanitarian ends. But until it is, like a prostrate ladder, it constitutes a trap for the feet of the unwary, rather than a pathway erected to higher things.

The book, therefore, is merely an attempt to fit together (1) the Newtonian mechanics, (2) the doctrine that heat is mode of motion, and (3) the dozen or so well known facts of thermal action, into a consistent whole which may serve as an engineer's idea of heat and heat-action. It was originally prepared for publication in the periodical press, and some of the earlier portions appeared, in preliminary form, in the columns of *The Engineer* (London). Some traces of this genesis may be noticed in the pages of the book.

The writer wishes to acknowledge his indebtedness to the Sheffield Scientific School, of Yale University, for helpful facilities for work.

NEW HAVEN, CONN., July, 1909.

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ENERGY

CHAPTER I.

Mechanical Energy.

The muscular system of our modern body politic is its array of energy-producing machines. Man has magnified his own tiny energies with power borrowed from nature. His land-carriers put to scorn the elephant; his ships make pygmies of the whales. Take from mankind to-morrow this artificial multiplication of its abilities to "overcome resistance," and within a month its ranks will be decimated by starvation. Within a decade it will have become a mob of howling beasts, fighting for the insufficient means of existence—slipped back centuries in growth of civilization as well as of population.

Yet to-day, in spite of this overwhelming importance of energy in modern community-life, there exists no idea of its nature more exact than the general one that it *is* an aid in the overcoming of resistance. Force times space, or mass undergoing acceleration, equals energy. Thus far we appear to proceed coherently. But it is not far enough, when the chief business of an increasing fraction of the human race is the transformation and transportation of energy—mechanical, hydraulic, electrical, chemical, thermal, etc.; and even in going that far we quite lack, it appears, a complete agreement between the authorities.

Then, again, the bulk of all this current supply of energy to the human race is obtained in the form of heat. Yet as to what heat is we know virtually nothing. It is commonly described, in an attempt to explain its nature, as a "mode of motion" between the particles of the hot body. But an explanation of an obscure thing, in order to help rather than hinder the understanding, must speak in terms of familiar things; and when this "explanation" of heat in terms of mass and motion is once examined, it is found to contain two elements which are even more unfamiliar and obscure than is heat itself.

The first of these elements is that "perfect elasticity" which must be attributed to the constantly rebounding particles upon the molecular theory of heat. For, since heat continues indefinitely in a hot body, so long as none is abstracted, each rebound must occur millions of times per second, with perfect elasticity, in order that the continuous existence of heat may be explained mechanically.

Yet such a thing as a rebound, after collision, with perfect elasticity, is unknown in nature. In fact, it is one of the fundamental doctrines of thermal science that motion *cannot* occur in nature without some loss of energy inelastically, in either friction or impact. If there be in nature such a thing as perfect elasticity, human observation has never yet discerned it. Therefore, the "mode of motion" explanation of heat is an explanation of one thing, namely, heat, which, albeit mystical in its nature, is yet familiar to every schoolboy, in terms of another thing, namely, perfect elasticity, which no one has ever yet known to exist anywhere. The only thing plain about such an explanation is that it doesn't explain.

The other element of common sense which is wanting in this alleged explanation of heat is an exact and familiar idea of mechanical energy itself. It is of no use to explain heat as a microscopic form of mechanical energy if we do not know what mechanical energy is. The term mechanical energy, it is true, is used most familiarly by engineers; yet, astounding as the fact may seem, it is nevertheless true that there exists to-day no agreement between the authorities as to just what mechanical energy is. The statements concerning it which are rife among the engineers, the teachers of engineering and their text-books, can all be reduced quickly to either absurdities or approximations. To "explain," therefore, that heat is a "mode of motion," when not one student of engineering in a thousand has ever been taught what modes of motion are possible in nature and what are impossible, is also a procedure of doubtful utility.

Nevertheless, there is no escape from the overwhelming and rapidly accumulating evidence that heat is capable of a real and true explanation, as an intricate form of mechanical energy. But progress can be made in this idea only by a careful investigation, first, of what mechanical energy itself truly is, in nature; and, secondly, of what conclusions should be pointed

therefrom as to the nature of heat, in natural common sense. Fortunately, every student of engineering science is equipped for both of these tasks, although few may have been led to perform them. A knowledge of Kepler's and Newton's laws of motion and force, a little analytical geometry, and enough of the concepts of the calculus to permit thinking of millions of molecules at once, without becoming confused as to what they may and may not do—this is all that is required.

Mechanical Energy. Mechanical force—that is to say, force manifested in such a way that the human understanding can follow its origin, dimensions and destination—is found in nature, exerted through space, in *only two* elementary ways. The first is when gravitation, which acts at all times between each two portions of matter in the universe to hold them *together,* finds chance to move them and to destroy their relative *separation.* Such action is called a manifestation of "potential" or "positional" or "space" energy, visible most familiarly in the falling of weights. The other instance occurs when either of the forces which everywhere and at all times tend to hold each two bodies in the universe both *together* and *apart* finds chance to produce or destroy their relative *motion.* Such action is called a manifestation of "kinetic" or "accelerative" or "motion" energy. It is visible in pure mechanics only in the action of centrifugal force.

It is to be noted carefully—much more carefully than the text-books require—that it is neither space nor motion alone which constitutes energy, but *change* in space or motion. A suspended weight possesses no energy if it never can fall. A flying cannon-ball can overcome no resistance if nothing ever interferes with it to stop it.

But there is another fact which is of even greater importance in these definitions, and which the text-books quite omit altogether. This is that in any energetic manifestation there are always involved at least *two* bodies. No single body may ever possess energy. When it is remembered that all of the commonly taught mathematical expressions for energy include but a single symbol for mass, and that in general terms energy is universally described as an attribute of mass, this statement cannot be too strongly emphasized. Indeed, the only proper way to state this idea is to say that energy can exist only where mass

is *not*—namely, *between* two mass-portions. It is just as broad a truth to say that mass and energy cannot occupy the same place at the same time as it is to say that two portions of matter cannot do so.

To explain more in detail: A ton of water in a mill-pond, we say, possesses energy. That is, it will overcome resistance in its fall toward the tail-race But why will it fall, and whither? What gives it its weight and energy?

Its separation from the earth gives it its energy. So soon as it succeeds in reuniting itself with the earth its energy is gone. The energy cannot be said to belong to the water, for without the propinquity of the earth the water would have no energy. It cannot be said to belong to the earth, for all the earth's gigantic mass would be inert and helpless to perform work were it a truly solid unit, with no fragments split off, like the water, to help it embody energy. And, finally, the energy cannot be said even to belong to the earth and water together, because both earth and water might be present—as is the case, indeed, in the great oceans—without embodying any potential hydraulic energy, because there exists no *separation* of earth and water, vertically; that is, there is no *head*. It is in the special relationship existent *between* the earth and mill-pond water that the energy lies, and not in either one, nor in both together. Literally, it lies in both apart.

The same statements hold true of kinetic energy. There is no energy in a flying cannon-ball, for instance, unless there be something to stop it. It is only in its stoppage that the projectile can overcome resistance; and it is only a second mass-portion which can do any stopping. No mere geometric point, nor co-ordinate axis, can arrest a moving mass. To refer the motion of any mass to such a geometric base, as a measure of its energy, is an error so fundamental that it has adulterated our entire science of energetics. There is only one base of reference for the energy of any mass-pair, and that is its common center of mass.

Kinetic energy, therefore, also consists of a *relationship between* two masses. It belongs to neither mass alone, nor even to their aggregation alone, but to their aggregation *when subdivided into separate portions* by relative motion between them.

In mechanical engineering these facts have long been lost

sight of (although long known), for two reasons. In the first place, one of the two mass-portions, in engineering problems, is always the earth; and the earth possesses so gigantic a mass, in proportion to that of any body of engineering magnitude, that it supplies an apparently fixed base of reference. Moreover, its great mass remains always constant, so far as engineering instruments can perceive, thus leaving the much smaller masses of our cannon-balls, railroad trains, etc., as the only variables.

In molecular mechanics, however, it is not only quite possible, it is altogether probable, that the various mass-portions are of the same order of magnitude; or, at least, if there be all classes of magnitudes, that there are many of each class. Under such conditions it is of prime importance to remember that there are two mass-members, at least, in each unit of energy, and that one of them is just as likely to be a variable as the other.

The second way in which the special conditions prevailing in engineering practice have warped its concepts of energy away from the truth is seen in the common idea of potential energy as synonymous with "up and down." To a student of purely mundane forces this is, of course, natural; but when one has studied true mechanics long enough to get away from the special conditions of the earth's surface, it is appreciated that potential energy refers only to "together and apart." There is no true "up" nor "down" in nature. It were well, for instance, as we stand by the brink of a Niagara, awed by the thunder of its action and marveling at even that slight modicum of its energy which the power-houses succeed in catching, to remember that almost beneath our feet the even more stupendous falls of the Zambesi, of more than twice the height and certainly equal power of Niagara, are thundering their watery masses *in exactly the opposite direction* from those before our eyes.

Yet in either case the action is the same. The energy lies not in the water of either cataract; it lies in the relationship existent between water and earth. And of this relationship no concept can be had by speaking of "up and down." It is the relative *separation* of earth and water, above the falls of either river, which constitutes the energy; and the only adverbs which will cover this idea are "together" and "apart." All portions of matter in the universe tend, by gravitation, to fall *together*, or to consolidate. All of them also tend, by centrifugal action, to fall

apart, or separate, or disintegrate. Sometimes, as on this earth's surface, the former tendency is overwhelmingly greater than the latter, and the latter is therefore lost sight of. But in many other places it is the centrifugal tendency which overwhelms and obscures the centripetal, so that the latter is forgotten. Such is usually the case in the so-called permanent gases, which tend always to expand indefinitely.

But for our present purpose, viz.: A true statement of principles such as may fit all cases, all that is necessary is to keep both facts constantly in mind: that at all times and places in nature both tendencies, the congregative and the disgregative, are at work, in opposition to each other—sometimes one and sometimes the other prevailing.

One of our national American humorists of a generation ago, in promulgating the manifold attractions of his world-famed "show," advertised the exhibition of one of the Siamese twins—"the only one which had ever been successfully separated." Not that the one had died, but that it had not yet been separated. The absurd humor of the remark never needed an explanation. And yet the two great twin forces of nature—the centripetal, or gravitational, and the centrifugal; the congregative and the disgregative—have ever been solemnly presented to our students of engineering, at different and disconnected times and places, as if each were the only one which had ever been separated from the other. They are taught as if each were an independent natural phenomenon.

As a matter of fact, the action of gravitation is always and insistently to combine and unify mass; and if it had its way unhindered the universe would soon become a single solid of infinitesimal dimension and infinite density. Centrifugal force, on the other hand, always and insistently tends to separate and diffuse mass; and if it had its way unhindered the universe would soon become a gas of infinite volume and infinitesimal density. But neither thing happens. So far as we can see, the mean volume and the mean density of the universe both remain constant. The simple explanation is that centripetal and centrifugal forces are *always* paired, in counterbalance. Sometimes one prevails, temporarily and locally, and sometimes the other. *But neither is ever absent.* Neither is ever completely free to act. The existence of either one alone is unknown in nature, is in-

conceivable to the naturally taught mind, and should never be mentioned to the student as a natural possibility.

Of these two fundamental mechanical tendencies let us consider first the centripetal one.

It may be regarded as the prime fact of nature that, except when an excess of motion disguises the fact, all things are obviously bonded together by a mutually attractive force. The "law" —or, better, the *fact*—of gravitation is simply the sublime condition that each two bodies in the universe, of whatever sort, at whatever places and at all times, are drawn toward each other by a mutual tie of affection—an affection so true and unvarying that to liken it to mere human affection, which is always partial and fickle, were belittling it indeed. This tie can never be broken, although it can be stretched indefinitely.

The old saw has it that it takes two to make a quarrel. Well, it takes two to make a bond of affection, just as well; and it takes two mass-portions to exert a gravitational attraction. As Newton defined the law, more than two centuries ago, the force between each pair of masses varies, first, directly as the product of their masses, and, secondly, inversely as the square of their distance. Stated mathematically this becomes

$$\text{Force} = c\ M_1 M_2 \frac{1}{S^2}, \tag{1}$$

wherein the M's represent the masses concerned, S their distance of separation and c a constant. When the masses are measured in units, each of which weighs 32.16 pounds, on the earth's surface, and when S is stated in feet, the value of c, in order that the force shall also read in pounds, becomes about 0.000,000,0343, or one divided by about twenty-nine or thirty millions.*

If, in this formula, M_1 should be stated as the mass of our earth and S as its radius, $c\ M_1 \div S^2$ would reduce to 32.16 pounds; or the gravitational force exerted by any unit mass at the earth's surface would be 32.16 pounds. If M_1 and M_2 were a pair of steel plates, one inch thick and about twenty-two feet square, hung up vertically in chains in contact, face to face, it would require a force of about one pound to overcome their mutual gravitational attraction and pull them apart.

*These figures follow Professor Poynting, as quoted by Professor J. J. Thomson in *Engineering* (London), March 19, 1909, page 397.

This may seem like a very small force. But it is only because either one of the mated plates, pulling upon its brother, is a very small thing in comparison with the earth which is pulling upon them both. It is also to be remembered that the force increases, as the masses grow larger, by their product, and, as the distance grows smaller, by the reciprocal of its square. Thus it becomes plain how this force may become sufficient, upon occasion, to hold in position the gigantic heavenly bodies, on the one hand, or so to bind together the minute particles of steel, on the other, that a stress of 100,000 pounds per square inch cannot pull them apart.

But force without space does not constitute energy. It is only as the separation *alters* that energy appears. Multiplying the force, therefore, by a small change in distance (dS, in the calculus) and integrating, there results

$$\text{Potential Energy} = E_p = c\, M_1 M_2 \left(\frac{1}{S_o} - \frac{1}{S}\right) = c\, M_1 M_2 \left(\frac{S - S_o}{S S_o}\right) \quad (2)$$

wherein S_o is any original distance of separation and S any other. As in Equation 1, if S be measured in feet and M in units weighing *g* pounds the result will be a value for E_p in foot-pounds.

If, further, the quantity $S—S_e$ be given the symbol *h*, Equation constant, and if $S—S_o$ be very small in proportion to S, then $c\,M_1/SS_o$ may be taken as a constant and given the symbol *g*. If further, the quantity $S—S_o$ be given the symbol *h*, Equation 2 becomes

$$E_p = g\, M_2\, (S - S_o) = Wh \quad (3)$$

which is the formula for potential energy more familiar to engineers than the correct one, Equation 2. But Equation 3 now appears in its proper light, viz: as a mere approximation which suffices in accuracy under certain assumed special conditions only.

The same general aspect of the situation applies also to the question of kinetic energy. If a moving body be accelerated, either positively or negatively, force is manifested and energy developed. But to this phenomenon the participation of at least two bodies is essential. No moving body can be accelerated except by reaction against a second body. We have Newton's own word for that. It is quite erroneous to say that any moving body possesses energy by reason of, and to the extent of, its

visible motion. Relative motion between two bodies can be perceived visibly quite independently of their relative mass. We can measure the motion of distant suns from a base of observation, to wit, the earth, so small that the sun in question would never perceive it if it hit us and wiped us out of existence. But the measure of the kinetic energy between two moving bodies depends altogether upon the mass of the point of reference. For a moving body possesses energy *only to the extent that it can be stopped.* And that extent is settled, not by the original body alone, but by the mass-relationship existing between motor and arrestor.

If any mechanic doubts this statement, let him try to forge a bolt upon an anvil made of wet clay, which cannot bring its entire mass promptly to the job of arresting the hammer. Or let him try it even with a steel anvil, but one having a mass only equal to that of his hammer. He will find that his hammer, and the muscles which yesterday drove it with effective energy, to-day are powerless. The muscles are vigorous and the hammer-head lively; but action can be no greater than reaction. As the anvil reacts, so only may the hammer act. Motion may be there, but not necessarily kinetic energy.

These ideas may be reduced to mathematical expression by starting with the empirical equation of motion,

$$\text{Force} = \text{Mass x Acceleration.} \tag{4}$$

From this there results, by algebra which need not be reproduced here, the following fundamental expression for the kinetic energy of any mass-pair:

$$E_k = \frac{1}{2}\ \frac{M_1 M_2}{M_1 + M_2}(V^2 - V_o^2) \tag{5}$$

wherein the M's are the two mass-portions (measured in units weighing 32.16 pounds at the earth's surface), the V's are the original and final velocities, in feet per second, respectively, and E_k is the energy in foot-pounds.*

* The reader may be assisted to connect these fundamental formulæ for energy with his more familiar engineering concepts by the following:

Let Fig. A represent two bodies, M_1 and M_2, having a common center of mass at C. Let the bodies possess a motion toward or away from C

If M_1 be very large in comparison with M_2 the fraction r
duces to approximately the value M_2. If, further, $V_o = o$, t
expression becomes

$$E_k = \frac{1}{2} M V^2 \qquad ($$

which is the special approximation which the engineering pr
fession generally regards as the true fundamental equation f
kinetic energy.

So long as the mind confines itself to engineering probler
this special approximation serves as well as the exact expressic
But the text-books certainly ought never to leave it to the mag
zine press to inform the student that it *is* a special approximatio
instead of the true article. And when this special approximatic
is carried into the problems of molecular energy, where it
applied as the sole available concept of kinetic energy, this b

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of v_1 and v_2 respectively. Then, from the law of conservation of m
mentums,

$$M_1 v_1 = M_2 v_2 \qquad ($$

and $$v_1 + v_2 = V \qquad ($$

wherein V is the relative velocity between the two bodies.

If this relative motion be opposed by a force of magnitu
$M_1 a_1$ ($= M_2 a_2$), acting upon each mass and reacting upon the other,
will be destroyed at a rate of negative "acceleration" equal to

$$A = a_1 + a_2 \qquad ($$

wherein a_1 is the change in velocity per second measured between M_1 a
C, a_2 is that measured between M_2 and C, and A is that measured betwe
M_1 and M_2.

When the motion has been entirely overcome the opposing force w
have been overcome by the body M_1 through the distance $\frac{1}{2}$ v_1, and l

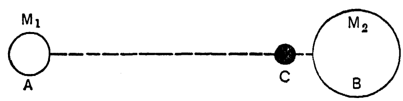

FIG. A.

the body M_2 through the distance $\frac{1}{2}$ v_2, or over a total distance of $\frac{1}{2}$
The work performed will have been equal to the force times the distan
in the case of each body, or, together,

$$\text{Work} = M_1 v_1 (\tfrac{1}{2} v_1) + M_2 v_2 (\tfrac{1}{2} v_2) \qquad ($$

From Equations a and b, $$v_1 = \frac{VM_2}{M_1 + M_2}$$

and $$v_2 = \frac{VM_1}{M_1 + M_2}$$

The substitution of Equations e and f in Equation d gives

$$\text{Work} = \tfrac{1}{2} \frac{M_1 M_2}{M_1 + M^2} V^2 \qquad ($$

for the special case where the relative motion is completely destroyed.

of careless neglect becomes an egregious, fundamental error, which has misled many an able mind which lacked the time needed to dig out the straight of the matter.

The significance of the above, aside from some most important conclusions which will be drawn from it in later pages, amounts to this:

1. Energy exists only *between* bodies, and never *in* them. No single body, by any quality assignable to it as a unit, can ever possess energy.

2. The energy frequently spoken of as internal to a "single" body implies that the body in question is not a single unit, but a swarm of tiny particles, each separate from its neighbors, yet too small to be seen separately, *between* which exists the energy said to be "internal" to the body.

3. The relationships between mass-portions which constitute energy may be either of two sorts, viz: *space*-relationships and *motion*-relationships, called potential and kinetic, respectively.

4. Neither relative space nor relative motion themselves constitute energy, but only *change* in either space or motion. Therefore, every true expression for energy must contain the difference between a greater and a lesser value for space or motion, as the case may be. And it is a general fact of nature that nowhere is the smaller measure of either space or motion ever known to become zero.

5. Every true expression for energy must contain, for its mass-factor, the arithmetical product of the two quantities of mass concerned.

6. The elementary or unit mass factor in energy is not a unit of mass, as is now universally taught, but the unit *mass*-PAIR; that is to say, a pair of mass-portions, each member of which is one unit of mass.

7. The ability of different mass systems to embody energy is proportional, not to the total mass of each, but to the *square* of that aggregate mass. This is necessarily true of space-energy, but may or may not be true of kinetic energy, according to the nature of the motions contemplated.

CHAPTER II.

FREE AND VIBRATORY ENERGIES.

In the preceding article energy was defined as a change in either the space-relationship or the motion-relationship between two or more bodies. It is necessary now to see what sorts of space and motion relationships are possible in nature.

Let Fig. 1 represent an ordinary pendulum attached to two A-frame supports on the earth's surface. Here exists an energy-system in which both space and motion relationships occur. Moreover, assuming the pendulum to be in motion, there is a constant change of both. Finally, there are present the two separate mass-portions which were stated in the preceding paper to be essential, between which the energy is embodied. One mass-portion is the pendulum-bob M_1 and the other is the earth M_2.

If M_1 be held stationary at A the system exhibits space-energy only. If it be released at A, however, the pendulum swings toward C, speeding up as it goes. But at C it has reached its maximum velocity, and beyond that point it slows down until B is reached, where it stops, reverses and repeats the process in reversed order. In this simple and familiar phenomenon is exhibited one of the broadest and most impressive principles of natural action, viz:

THE CONSERVATION OF ENERGY.

For if the space-energy lost between A and C be carefully measured, and also the motion-energy gained, the two will be found to be equal. Expressed mathematically,

$$\frac{1}{2}\ \frac{M_1M_2}{M_1+M_2}(V_c{}^2-V_a{}^2)=c\ M_1M_2\left(\frac{1}{S_c}-\frac{1}{S_a}\right) \qquad (7)$$

wherein the *a*-subscripts refer to conditions at either A or B, and the *c*-subscripts to those at C. This is the fundamental equation for *energy-transformation.*

Fig. 1, however, does not represent an ideal or perfect system, because the connection between the two masses is established by means of a nominally flexible cord or hinge, which in

actuality always involves friction. Nor may it be inferred from this that the words ideal and perfect refer, as they usually do when used in connection with heat-engines, or gases, to some imaginary and impossible conditions, never found in nature. For in mechanics, while there is no known instance of force

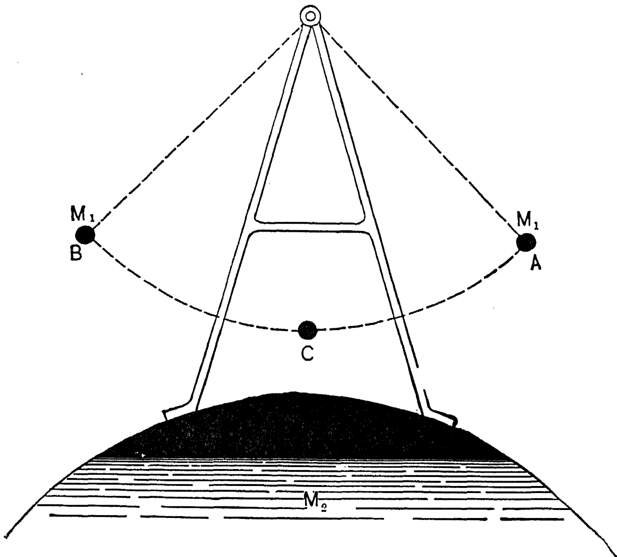

FIG. I.

being transmitted from body to body *by contact* without some loss of energy in friction and impact, yet it is common for such to occur by *action at a distance*. Indeed, there exist no two bodies in the universe, according to Newton, between which force is not thus perfectly transmitted, by gravitational attraction, and between which motion cannot occur under forceful control, yet without friction. This fact is the very foundation of our entire science of mechanics.*

*This statement takes entire cognisance of the resistance to the motion of celestial bodies through interstellar space which has been revealed by modern astronomy. The fact of this resistance upholds all the positions taken here and in later pages, as to our fundamental concepts of true mechanics, celestial or molecular. The laws of Newton and Kepler, and the fundamental mechanical principles which have been deduced therefrom, are all based upon the assumption that the celestial bodies move through matterless space. They hold true only in that case. Indeed, our only idea of a "body" is a portion of matter separated from other portions of matter by true space, in which exists no matter. Later in these pages it is

The use of this gravitational action at a distance to link two bodies together into an energetic system similar to Fig. 1 is shown in Fig. 2, as a diagram of an energy-transforming system which would remain in operation indefinitely, with its energy perfectly conserved; because it relies solely upon "action at a distance," or force transmitted without friction. In Fig. 2 one of the masses, M_2 (the earth, if you please), is shown pierced centrally with a great well-hole—much as Captain Symmes imagined the earth

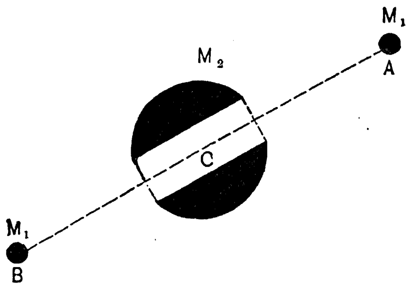

FIG. 2.

to be from pole to pole, in his long-ago famous "Symmes Hole" theory. Above the earth's surface and in line with this hole, as at A, is suspended the body M_1. Upon its release it will vibrate, quite as did the M_1 of Fig. 1, on either side of the point of closest approach to M_2 at C.

To this system Equation 7 applies perfectly. In this case, too, occurs good illustration of how the lesser distance of separation S_c never becomes zero, although at the point C the *geometric* centers of the two spheres happen to coincide. But in energetics

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stated, as a broad natural empiricism, that there is no such a thing as true "space" devoid of matter—that some degree of mass, pressure, temperature, etc., exists everywhere. This is upheld by the fact of resistance to celestial motion showing that the celestial bodies move through, not space, but attenuated matter, probably interspersed with small solid bodies. It will also be pointed out that there exists in nature no such a thing as true matter; that is, matter devoid of space.

All this does not affect the fact that we are now two centuries gone on a course starting from the concept of true matter and true space, as contrasted absolutes. Every boy's experience, every student's training in elementary mechanics and every engineer's instinctive judgment are based upon this same idea, which is also the foundation of the Newtonian mechanics. The writer is merely insisting that, since the doctrine of all mechanics as founded on the Newtonian concepts, and of heat as being "a mode of motion," is promulgated universally in all engineering schools, its consequences must be faced with consistency.

it is only mass which counts, not geometry. Although the geometric centers may coincide at C, the two masses do not. After M_1 enters the geometric boundaries of M_2 their real separation decreases only gradually, until at C it is a minimum, but not zero.

It happens, however, that the case illustrated in Fig. 2 never occurs in nature. It is not that the major mass-portions of the universe neglect to possess convenient openings for the passage of the minor mass-portions through them. The trouble is that in nature the chances are infinitely against any pair's ever possessing a motion, when in the condition A, which is directly alined with their mutual centers. The thing is conceivable geometrically, but it is even less than probable. When the natural causes of different forms of motion are investigated, it will appear that such motion is impossible. Motion developed naturally, rather than in the imagination, will be directed at some appreciable angle with the mutual axis, as at A in Fig. 4.

In any natural case the mutual motion will be likely to assume the form portrayed in Fig. 3—to consider the simplest case first. Here the bodies are shown as revolving about each other, and also about their common center of mass at C. Each body describes an elliptical orbit about its mate, and also about the point C. For a moment's consideration will show that, whatever form of orbit one body follows around the other, the other must likewise describe about the one; and if the mass-center C be regarded as the fixed center, each body will describe about it similar orbits, having radii inversely proportional to the mass in question.

The situation is much as if a boy fastened a large cannon-ball on one end of a stick and a smaller one on the other. He can then twirl the stick by holding either end in his hand, regarding that as fixed while the other end moves, or he can hold the middle portion of the stick in his hand, at the center of gravity, and twirl both ends at once about that. But in the case of the boy and the stick, his hand is attached to the earth, and may be regarded as fixed; whereas in the case of the two free mass-portions, if we are to study their own interaction, independently of all other masses, there is no fixed base to refer anything to. Should the boy throw the stick into the air, however, for a brief period it would act freely as an independent, free mass-system, and during that time the two cannon-balls

would revolve, each about their common center of mass. For that reason it is proper to consider the energetic action of the members of a pair only in reference to their *common center of mass.*

In any such an elliptic orbit the condition of greatest separation, as at *ab,* is called the *apastron* of the orbit, while that of least separation, as at AB, is called the *periaston.* At apastron the energy existent between the two is chiefly space-energy, but there is also a little motion. At periastron the energy is chiefly

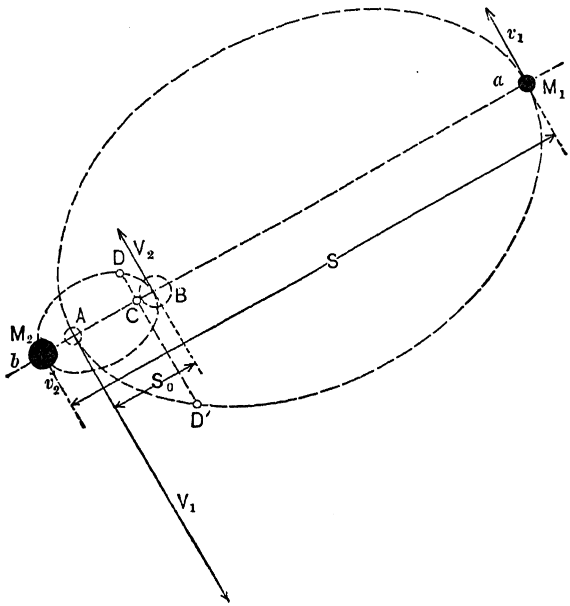

FIG. 3.

motion-energy, but there is also a little space present. The equation which connects mathematically these maxima and minima of space and motion is Equation 7; and in it, in all truly natural cases, neither V_0 nor S_0 may ever be regarded as becoming equal to zero—as will be made plain as the argument proceeds.

For Fig. 3—or, rather, its more general form, Fig. 4—represents the only true element of mechanical action. For any such an element, in order to be an element, must be absolutely "free." That is to say, it must be considered independently of all other

masses. Yet it must be capable of containing energy. Therefore, it must be, not an ultimate or indivisible unit of mass, but a *mass-pair,* an elementary mass-pair.

All action between solid bodies in contact, on the other hand, such as is familiar to all engineers in their machines, is not purely mechanical. It is always partly thermodynamic, in so far as heat is being constantly developed by friction, and partly a special case of pure mechanics, in that the body is "constrained" rather than free; that is, it is handling energies which are transient through it from without, which are independent of its own mass, and which are ultra-complex in their nature.

Fig. 4, on the other hand, is entirely general. It displays every possible form of pure and elementary mechanical action between two bodies, supplied with any original store of relative space and relative motion whatever, as at A; and it introduces no foreign element of dependence upon any other mass-system or form of energy whatever. Nor does it introduce any unnatural assumptions. Without stopping now for the proofs, it may be said that any such a case must resolve itself into the mutual revolution of the bodies about each other in an orbit which follows some of the *plane conic sections*—either the *hyperbola,* the *parabola,* the *ellipse,* the *circle* or the *straight line.*

Further, it can be shown that if the original energetic condition of the pair at A M_2 be known, by knowledge of the distance d between the two, the velocity v of their relative motion, the angle ϕ existent between d and v, and the two masses M_1 and M_2, then the nature of the orbit is known, and also its dimensions. Both are best expressed in terms of the distance D between the two when separated by a radius normal to the major axis XX′ of the orbit, the velocity U at that point, and the angle α between D and U. The mathematical relationships between all these quantities will be discussed later.

Of all these apparently varied forms of motion only two, the hyperbola and ellipse, are probable forms. For between any two masses, at any given initial distance, there may be an infinite number of directions and magnitudes of velocity which would result in hyperbolic motion, and another infinite number which would result in elliptic motion. But there is only a single direction of motion which would result in a straight-line orbit, such as that of Fig. 2; and there is only one other direction of motion,

with a particular velocity into the bargain, which would result in circular motion. A parabolic orbit could likewise result from only a single direction of motion, coupled with its proper magnitude. Therefore the mathematical chances are infinitely in favor of all motion in the universe being either elliptic or hyperbolic in form.

The possibilities, however, are not even so complex as even this statement might indicate. Speaking mathematically, there is for all of these forms of orbit but a single, basic equation. This equation must involve some factor expressive of radial distance between the two, and also some factor expressive of either the

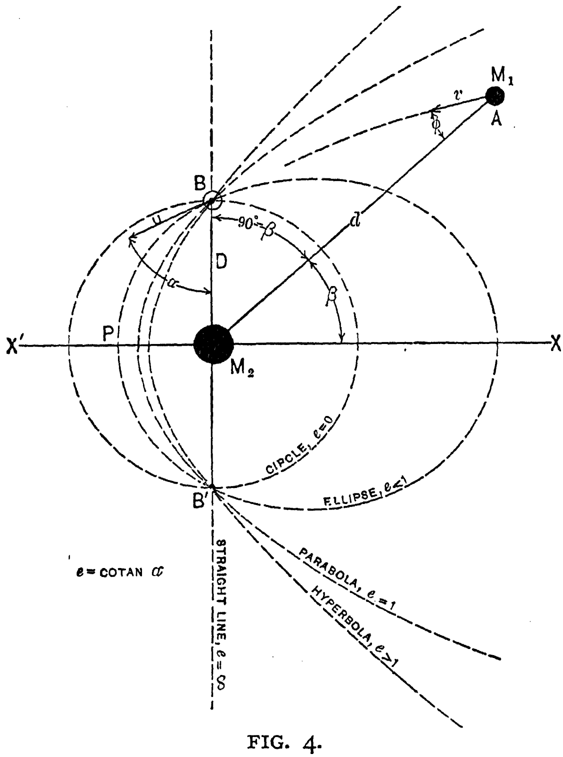

FIG. 4.

two quantities of mass or their periodic velocity of revolution. These will define the general magnitude of the system. But besides these factors there must also be present, as an indicator of the *form* of orbit, another which is called the *eccentricity*. And the eccentricity of orbit, it will be found, is a factor of

fundamental importance in the determination of whether energy-transformation is to occur or not.

Without attempting to enter into any discussion of the mathematical definition of eccentricity, which is usually given the symbol *e*, the following characteristic facts should be noted carefully, as indicating sufficiently for our purposes its general nature:

1. If the eccentricity be zero the orbit is a circle;
2. If the eccentricity be greater than zero, but less than unity, the orbit is an ellipse;
3. If the eccentricity be unity the orbit is a parabola;
4. If the eccentricity be greater than unity, but finite, the orbit is an hyperbola;
5. If the eccentricity be infinite the orbit is a straight line.

Of all possible cases, therefore, the circle constitutes one extreme and the straight line the other. Mathematically speaking, they constitute mathematical limits, with the chances infinitely against their ever being attained in nature. But, speaking naturally, this means that they never occur. Both zeros and infinities are unimaginable, as natural phenomena. They have never been observed, and, so far as the human mind may predict, they never will be observed. It is one of the heaviest indictments to be brought against our present methods of teaching natural science that its text-books are so filled with reckless use of zeros and infinities. It is not that zeros and infinities should never be employed, but that they should always be specified, before using, as natural impossibilities—as is always done, for instance, in specifying the exclusion of friction, thermal conduction, etc.

As for the parabolic orbit, that plainly stands as the dividing case between the ellipses below and the hyperbolas above. It itself, like the circle and the straight line, is infinitely improbable of occurrence, in any permanence of form. But whereas the chances are infinite against truly circular or straight-line motion ever being attained, even instantaneously, the condition of parabolic motion must be at least crossed, and therefore existent instantaneously, in transition from elliptic to hyperbolic motion. For the parabolic condition is like a dividing line between two areas: having no dimension, substance or reality itself, it must

nevertheless be encountered and crossed by substantial realities, in their passage from one territory to the other.

But, it will naturally be asked, what have all these astronomical illustrations of "free" bodies to do with engineering mechanics, when no engineer or mechanic ever saw anything in his work which acts in that way? The answer is that, in spite of the strangeness of the idea, this is the *only* way in which any portion of matter ever really does act, when it acts purely mechanically. In some departments of mechanics, chiefly in gunnery, the projectiles which form the chief subject of study do come very nearly to acting in this way. The only assumption which has to be made, in reducing their ordinary action to a true, or "free," or natural basis, is that the friction of the atmosphere be absent. Then their paths are commonly said to be parabolic. But this is a statement which is only approximately true. It is made because it is simple and easy to assume that the lines of gravitational force which radiate from the earth's center are, for small portions of its surface, virtually parallel; and also because the result is not far enough wrong to upset calculations.

But for the special purposes of our argument it is far better to keep in mind the exact truth—which is simple enough so long as we do not try to make any detailed calculations—namely, that all cannon-balls, base-balls, foundry-drops, etc., are in reality following elongated elliptical orbits passing closely around the center of the earth. But of this orbit the only portion which we can see is the apastron tip. The remainder of the orbit never is traversed, because collision with the solid surface of the earth breaks up the phenomenon, in a dissipation of energy in thermodynamic, rather than mechanical, action.*

But even to get such a slight peep as the above at pure mechanical action on the surface of the earth, we have had to

*The foundry-drop is ordinarily regarded as falling toward the earth in a straight vertical line. Yet, in reality, as it leaves the latch it possesses a tangential velocity, parallel with the earth's surface, of over fifteen hundred feet per second, due to the earth's rotation. If we could imagine the mass of the earth as suddenly concentrated at its center, or within a sphere about ten miles in diameter, the foundry-drop would be free to describe an elliptic orbit passing around the center of the earth at a distance of about seven miles, and exhibiting at periastron a tangential velocity of 170 miles per second. Midway toward the center of the earth this ellipse would spread out to a conjugate diameter of over one hundred miles. Yet this illustration is the nearest approach to rectilinear motion which we can produce in the engineering arts!

make the never true assumption that the friction of the atmosphere were absent. Yet in all the mechanical actions which are even more familiar to the engineer, such as the interaction of solid machine-parts, see what even more wholesale assumptions have to be made, in order to weed out the non-mechanical phenomena of friction and impact and get a glimpse of the pure mechanics behind them! It is scarcely worth while to try. And yet it is none the less true that there is never a hammer-head, piston or shuttle started into motion that it does not *try* to follow an elliptic orbit about the earth's center; from which it is constrained only by constant supplies of energy from without, in the form of solid forces not dependent upon the masses transmitting them, which forces and energies are called "transient."

For taking the time to study and understand these free or natural tendencies of mass, which get so little chance to display themselves here upon the earth's surface, there are two reasons. One of these reasons is the fact that they constitute the only true mechanics, from which all engineering happenings are but special departures and for which only approximate statements can be made. It is therefore the soul of true education to teach these exact truths first, displaying their useful applications afterward in their proper aspect.

The other reason is that no concept of heat as a mechanical phenomenon can possibly be attained without them; for in the mechanical interaction between the particles of a body, by which we must now attempt to explain not only heat, but surely also chemical action and probably also electrical and magnetic energies, there can enter no friction nor impact. The action must be purely mechanical. There can be no dissipation or degradation of energy into heat, because it is heat itself which is being considered. Nor, as stated in the last paper, is it any explanation of the puzzle to specify that the particles shall be perfectly elastic; because perfectly elastic *matter* is unknown in nature. It is only *space,* devoid of solid contact, which is perfectly elastic in nature; and the first task in the comprehension of heat, therefore, is to comprehend thoroughly this "free" department of natural action, in which friction and impact are unknown and unimaginable.

The task of the theorist, in fact, is not so much to explain action at a distance in terms of action by contact, as is so often

assumed, but fairly the reverse. The obscure and intricate happenings, which, in our ignorance we lump together under the convenient blanket-term "contact," involving always interchanges of pressure and heat, and often mechanical, chemical and electrical energy, none of which we understand, can be explained simply and clearly only when they are reduced to terms of action at a distance. For the latter demands no "explanation," or reduction to terms of something else. It is beautifully simple, having been defined in an elementary algebraic formula by Newton two centuries ago. It is complicated by no questions of elastic pressure, thermal or electrical conduction, or chemical interaction, varying interminably with each new case of "contact." Centripetal gravitation, like its mate, centrifugal force, is one of the basic facts of the universe—more basic even than matter, the existence of which we infer from its gravitational and centrifugal action—and forms no proper food for further analysis until it shall appear to us in much more intricate guise than that which we inherit from Newton.

Our sole duty here is to recognize that "contact" is merely a convenient name for impact and friction, when we are discussing mechanics, for thermal conduction when we are discussing heat, for electrical conduction when we are discussing electricity, etc. "Action at a distance" implies merely the absence of any of these energetic transformations.

These, then, are the elementary laws of motion, when viewed from the standpoint of energetics rather than kinematics. Since they appear to be quite different from the laws of motion of Newton, with which every student is familiar, while apparently of an elementary importance equal to those of Newton, it is important to note that they are in reality the laws of Newton, but stated in combination with each other and with the laws discovered by Kepler some sixty years before Newton enunciated his law of gravitation in 1680. Newton's laws of motion, in their familiar form, constitute one exceedingly simple form in which the elements of mechanics may be expressed. The trouble with their form is that, in order to get each statement into its simplest possible form, a set of premises has been assumed which is peculiar to that particular statement only, and which

not only never occurs in nature, but which is directly inconsistent with some other of this same set of laws.

For instance, one of these laws of Newton asserts that "a body once in motion will continue in unchanging straight-line motion until interfered with by a force." But Newton's own law of gravitation declares that no body in the universe may ever dissociate itself so remotely from all others as to be free from interference by forces, and by an infinite number of forces at once. Therefore, unchanging or straight-line motion can never occur in nature. For some special problems of mechanics it has been useful to assume that it could. But the wholesale manner in which this special and temporary assumption has been permitted to usurp the place of a fundamentally correct and permanent principle of nature has undermined our entire understanding of the problem of energetics.

Again, for instance, "a body subject to a constant force will experience constant acceleration." But again, Newton's own law of gravitation declares that force can remain constant only so long as the separation between the bodies remains constant. But this is impossible, motion being assumed at all, except when the motion has the form of a circular orbit, at constant distance; in which case the motion and force would be at right-angles with each other, and mutually independent. In all other forms of motion the force must always be varied by the motion which it itself produces. Therefore, constancy of acceleration, whether positive or negative, is unknown in nature.

To understand properly Newton's laws of motion, therefore, they must be coupled with each other and with the laws of Kepler—upon which latter, indeed, they were founded. Kepler's laws are three in number, viz:

1. The natural path of free motion between two masses is one of the plane conic sections.*

2. The area swept over by the radius-vector connecting the

*Kepler, working with the planetary orbits alone as his material, and quite as a pioneer, stated this law originally as including only the ellipse. It is later learning which has broadened the statement to include all of the conic sections. A very simple and elegant proof of this law—depending, however, upon Kepler's Second Law—was published by Mr. Immo S. Allen, of the London Institution (Finsbury Circus, London, E. C.), in the *Scientific American* of July 10th, 1909.

two is everywhere equal for equal periods of time; or the "areal velocity" is constant.

3. The square of the orbital period (or time) of revolution is proportional to the cube of the major axis of the ellipse, divided by the sum of the two masses.

The third of these laws again illustrates the way in which fundamental error has been permitted to creep into the standard college text-book. Of all of the ordinary text-books in astronomy which the writer has happened to enter—not to find fault, but in a sincere effort to straighten out this tangled question of "what is energy"—only one mentioned the sum of the masses at all, as a factor in Kepler's third law. All others omitted it, making of the expression for the third law a special approximation as erroneous as is the formula ½ M V² for kinetic energy.

Indeed, until this single case was discovered, the writer was quite puzzled by the situation; for, according to all he knew of the elements of mechanics, the sum of the masses ought to appear in the statement of Kepler's third law. And yet here was standard text-book after text-book which made no mention of them! Was all that he thought he knew nonsense, or where else was the trouble? In his quandary his heart went out to all those unfortunates who may have made serious attempt to understand the general principles of energetic action from the mechanics taught in the colleges as a foundation.

If, now, all the laws of Newton be kept in sight at once, and those of Kepler with them, the elementary principles of all mechanical action may be restated as follows. In their literal form they apply only to an energetic system consisting of a single pair of bodies. In the sense that every portion of the natural universe may be—and, according to any philosophy founded on the Newtonian mechanics, must be—considered as made up of a large number of such pairs, with its distances, forces and motions all reducible to an equal number of components, one for each pair, they are universal in their application.

1. Everywhere is space. No two bodies may be conceived as coincident, nor any one body as occupying zero space. The "occupation of space" has always constituted the fundamental definition of "matter."

2. All space is relative, measurable only *between* bodies. Absolute space is as inconceivable as is absolute lack of space.

3. Everywhere is force. Freedom from finite force is unknown. No two bodies may ever become so widely dissociated as to reduce their mutual attraction to zero, nor so closely coincident as to raise it to infinity.

4. All force is relative. It exists only *between* bodies, and may never be imagined as exerted absolutely, independently of mass.

5. Everywhere is motion. Absolute rest, or fixity, is unknown and inconceivable.

6. All motion is relative. Motion is measurable and conceivable only *between* the members of a related pair. No single body, independently of all others, may possess motion. Absolute motion is as inconceivable as is absolute rest.

7. Constancy of either space, force or motion is unknown in nature. Space varies force, force varies motion, and motion varies space, all the time, between any and every two free bodies.

8. Neither straight-line nor circular motion is known in ature. The only path of motion which is natural, rather than maginary, hypothetical and superstitious, is either the ellipse or he hyperbola, with velocities varying as stated by Kepler and orces varying as stated by Newton.

9. In any energetic system the primary fact is the conancy, or indestructibility and non-creatability, of its mass. The rinciple of the *Conservation of Mass*, discovered first and eeded first, should certainly receive the title of FIRST LAW OF NERGETICS.

10. In any such a natural, free system there occurs periodlly, at each revolution, a reversed transformation of energy, om space to motion form and back, under the Principle of the *nservation of Energy*. Discovered only in 1837—after much eliminary investigation and partial knowledge—and not yet ly understood, this great natural principle is properly to be itled the SECOND LAW OF ENERGETICS, not the "First," as it now called.

CHAPTER III.

The Mean Energetic Condition and the Energy-Fund.

In the last paper attention was called to the fact that all "free," or natural systems of mechanical action might be represented by some one of the conic sections, such as were exhibited in Figs. 3 and 4. Of these, for the present purposes of discussion, the elliptical orbit, as shown in Fig. 3, will suffice as an illustration.

In elliptical motion, as was pointed out in that paper, there occurs a periodic energy-transformation at each revolution of the bodies about each other. At apastron space is a maximum and motion a minimum; at periastron the reverse is true. And at every point between these extremes the conservation of energy is maintained; the amount of either form of energy lost, below the maximum, is made good in the other form.

From these facts it is readily inferred that there must exist between the extremes *a* and A some intermediate point where this transformation of energy from space-form to motion-form, or the reverse, is just half accomplished. Such a point would constitute a true *energetic mean* between the two extremes. At that point half of the total range of potential energy will have been converted into or from the kinetic form, and half will yet remain to be converted.

Let us indicate the distance of separation between the two bodies when in this mean energetic condition by D, and their relative velocity by U. Then there must result, from Equation 7 for the conservation of energy,

$$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{M_1M_2}{M_1+M_2}(V^2-V_o^2)=\frac{1}{2}\ \frac{M_1M_2}{M_1+M_2}(V^2-U^2) \qquad (8)$$

and

$$\frac{1}{2}c\ M_1M_2\left(\frac{1}{S_o}-\frac{1}{S}\right)=c\ M_1M_2\left(\frac{1}{S_o}-\frac{1}{D}\right) \qquad (9)$$

These equations readily reduce to

$$U^2=\frac{1}{2}(V^2+V_o^2) \qquad (10)$$

and $$D = 2\frac{SS_0}{S+S_0} \tag{11}$$

In the case of elliptical orbits it is usual to represent half the major axis by *a,* half the minor axis by *b* and the eccentricity by *e.* In that case

$$S = a\ (1+e) \tag{12}$$

$$S_0 = a\ (1-e) \tag{13}$$

and $$D = \frac{2\ a^2\ (1-e^2)}{2a} = \frac{b^2}{a} \tag{14}$$

But $\frac{b^2}{a}$ is one-half of the *latus rectum* of the ellipse, or the diameter through either focus at right angles to the major axis. That is to say, *the mean energetic distance D is the vector or radius joining the two bodies when they are situated directly at right-angles to the axis joining apastron and periastron.*

In short—and this seems to be most important—the energy-transformation which is always a part of the revolution of two bodies about their common center of mass, is a function of *angular,* not *linear,* motion. No matter how eccentric the ellipse may be, it is always true that in each quadrant of its motion the energy is just one-half transformed—from extreme space or extreme motion to the mean energetic condition, or back.

There is one case in our own solar system, for instance, where one quadrant of the elliptic orbit, that from apastron to the mean energetic condition, occupies about four hundred years and covers a distance measurable in hundreds of millions of miles. Yet the next quadrant, from the mean energetic condition to the extreme energetic condition nearest the sun, transforming an equal quantity of energy, occupies only a little over an hour and covers a distance measurable in thousandths of the other. And there may be, of course, even more extreme illustrations of eccentricity of orbit than this.

In Fig. 3 this mean energetic position is shown at DD′, and in Fig. 4 at BM_2. In Fig. 4 the mean energetic velocity U is shown as maintaining the angle α with the vector, or latus rectum, D. It will be convenient to note, concerning this angle α for future purposes, that

$$e = \frac{1}{a}\sqrt{a^2 - b^2} = \text{cotan } \alpha \tag{15}$$

These statements and arguments, although they are expressed most simply in terms of the ellipse, could be established as applying equally to any of the conic sections, such as are shown in Fig. 4. In the case of the parabola and hyperbola, which have no apastron ends and no definite major axes, the case is complicated (as will appear later) by the presence, within the pair of mass-portions, of two funds of energy—the one which has just been discussed and another. For the present it will be quite sufficient to the student to accept on faith the statements that the discussion is founded upon principles which are general in their character, applying to all possible cases.

In general, then, if the original energetic relationship of any pair of masses whatever, such as M_1 and M_2 of Fig. 4, be known—by data as to their masses, their distance *d*, their velocity *v*, and the angle ϕ which the latter makes with the vector *d*—all the conditions of the orbit and the energy-fund embodied by the pair are known. The all-important question as to the *amount* of energy thus embodied, which is a very difficult one, will be resumed at a later point. What is of more elementary concern at present is the *form* of the energetic action; and this depends primarily, it is obvious, upon the eccentricity of orbit.

It is worth while to repeat, in a briefer form and with inclusion of the angle α, the list of the different (mathematically) possible types of orbit which was given in the preceding paper.

1. If $e=0$, $\alpha=90°$ and the orbit is a *circle*.
2. If $e<1$, $\alpha \begin{smallmatrix} < 90° \\ > 45° \end{smallmatrix}$ and the orbit is an *ellipse*.
3. If $e=1$, $\alpha=45°$ and the orbit is a *parabola*.
4. If $e>1$, $\alpha<45°$ and the orbit is an *hyperbola*.
5. If $e=\alpha$, $\alpha=0°$ and the orbit is a *straight line*.

It is therefore most significant that, throughout all this wide variation in values of *e* and *x*, and in diversity of form of orbit, the mean energetic condition should remain consistently at the position normal to the orbital axis. Whatever may have been the original distance of separation from which the two bodies fell together, or whatever may have been the angle swept over by the vector between the positions A and B, or whatever may be the speed and propinquity at which the bodies pass each other at periastron, it always holds true that, once the mean

energetic condition is reached and one-half the energy-transformation accomplished, at B, a further swing of just one quadrant is consumed in completing the other half of the energy-transformation, to periastron at P. After that a second quadrant is consumed in reversing this energy-transformation, to the mean energetic condition again, at B′; after which the cycle ends with a reversal of the angle AM_2B, whatever it may have been—a quadrant in circle or ellipse, or less than a quadrant in the parabola or hyperbola.

Should the line BB′ of Fig. 4 be considered as representing a plane normal to XX′, and $90° - \alpha$ as the *angle of incidence* thereto, the *angle of reflection* therefrom, at B′, will always be its equal.

The Energy-fund: Radial and Tangential Energies. If it be true that the original position and motion of the pair at any original condition, such as A, Fig. 4, defines all the conditions of the orbit, it should be possible to define in terms of them the total fund of energy existent in the pair. Mathematically speaking, this is possible—so far as it is possible to define an energy-fund in terms of any premises at all. But the equations which connect any original condition, such as A, with the conditions B, P, etc., are so cumbrous in form, when combined, that it is not practicable thus to define the energy-fund in terms of any original point. It must suffice, instead, to know that the connection between A and every other point in the orbit is rigid and exact. It is quite sufficient for present purposes to investigate the energy-fund in terms of the conditions B and P only, as bases.

The kinetic energy visible at B in the velocity U is divisible into two components, the *radial* and the *tangential,* respectively. To each of these must correspond a respective fund of energy, radial anl tangential in its nature. Now, these two funds of energy bear a marked contrast with each other, in many respects; and as this contrast runs through the entire question of energetics, it is worth while to discuss it somewhat carefully.

In the first place, space-energy can, of its very nature, exist only radially. It is impossible to think of pure separation between two bodies—which, if they be truly single bodies, must be regarded as geometric points at their centers—in connection with any idea of *direction* of separation. Moreover, it is not only

that direction of separation is unthinkable in this connection; it would have no effect if it were conceivable. For the force of gravitation operates equally in all directions, and radially only; and as space-energy is based directly upon this force, it too must be regardless of angular direction.

Now this energetic force of gravitation works always in one direction, of the two directions possible within each radius. It always urges the two bodies together. Urging the two bodies apart, at all times, is the centrifugal force; and this force is a function of the tangential motion-energy. The two forms of energy, spacial and kinetic, are therefore in energetic counter-balance, just as much as their forces are in dynamic counter-balance. When the space of separation becomes greater than normal, it prevails over the motion-energy and forces the two bodies into greater propinquity. When the motion becomes greater than normal—and this is always the result of the action just noted—it prevails over the gravitational attraction and forces the bodies apart. Thus these two forms of energy vibrate against each other, in stable equilibrium, about the mean energetic condition as a center.

Now this centrifugal force, although itself radial in direction, is developed *only by tangential motion;* and it is in permanent, not vibratory, equilibrium that the tangential component of the velocity U finds itself in counterbalance with the gravitational force. To explain, let us consider first the case of purely circular motion.

In this case the radial motion-energy is zero, and the spacial separation is constant; no energy-transformation whatever takes place and no energy is manifested radially. All points in the circle are equally mean energetic conditions. The balance between centripetal and centrifugal forces is perfect and permanent. That is to say, speaking mathematically,

$$\text{Radial Force} = \overset{\text{(centripetal)}}{c\, M_1M_2\frac{1}{D^2}} = \overset{\text{(centrifugal)}}{\frac{M_1M_2}{M_1+M_2}\cdot\frac{U^2}{D}} \qquad (16)$$

whence

$$\frac{c}{D} = \frac{U^2}{M_1+M_2}. \qquad (17)$$

In the second member of Equation 16 the reader may not immediately recognize the more familiar expression for centrifugal

force, $M\frac{V^2}{R}$. The latter, like the other familiar mechanical equations which have already been criticised, is a special approximation, omitting the sum of the masses; which is sufficiently accurate when one of the masses is so large that variation in the other does not appreciably affect their sum.

If, now, comparison be undertaken between two circular motions of the same mass-pair, one at a radius r and velocity v and the other at radius R and velocity V, the change in space-energy which would be involved in a passage from one to the other would be, from Equation 2,

$$E_p = c\, M_1M_2 \left(\frac{1}{r} - \frac{1}{R}\right). \tag{18}$$

The change in velocity, from v to V, which must occur simultaneously in order that centripetal and centrifugal forces may remain balanced and the orbit remain circular, will be given by Equation 17; or

$$v^2 = (M_1 + M_2)\frac{c}{r} \tag{19}$$

and

$$V^2 = (M_1 + M_2)\frac{c}{R}. \tag{20}$$

Therefore the change in kinetic energy involved must be

$$E_k = \frac{1}{2}\frac{M_1M_2}{M_1+M_2}(v^2 - V^2) = \frac{1}{2}c\, M_1M_2 \left(\frac{1}{r} - \frac{1}{R}\right). \tag{21}$$

But the last term of this equation is one-half of Equation 18, the potential energy involved in passing from radius R to radius r. The energy absorbed in altering the velocity is therefore one-half the amount, and of opposite sign, from that released in increasing the propinquity (if the second radius be considered smaller than the first). The net energy released, therefore, is the algebraic sum of the two, and is itself equal to the last term of Equation 21. In other words, when masses are brought into greater propinquity *at their mean energetic condition,* or *with circularity conserved,* or *in permanent equilibrium between centrifugal and centripetal forces* (for the argument applies equally to circular motion or to the mean energetic point of elliptical or hyperbolic motion), energy must be abstracted; just as it must be when they are allowed to fall directly together, vertically, with no tangential motion. But in the former case the

amount of energy thus to be abstracted is just one-half of that requisite in the latter case.

The addition to the circular motion of a radial component, producing elliptic or hyperbolic orbit, will not affect this universal equilibrium between tangential motion and mean energetic distance.

In the case of free, conic-section motion, on the other hand, with *energy,* rather than *circularity,* conserved, no energy need be abstracted at all, as the propinquity increases. Indeed, to attain, under a fixed mean energetic distance, a greater propinquity at periastron, energy must be *added,* and in such a way that it assumes the radial form. In other words, to produce a *permanent* consolidation of matter, tending to continue in stable equilibrium, energy must be abstracted. But to produce a temporary consolidation, which will reconvert itself promptly into separation or disgregation of matter, energy need not be abstracted. It even needs to be added.

Further, it is evident from Equation 17 that, as the propinquity increases (or the distance of separation decreases) the velocity must increase. Therefore, putting Equations 17 and 21 together, it becomes plain that when masses approach, with circularity conserved, *energy must be abstracted as the velocity increases*—which is just the opposite of what is ordinarily held to be a universal law. But in the case stated, it is to be remembered, it is not the radial, but purely the tangential, velocity which increases as energy is abstracted. This is one of the many ways in which the radial and tangential forms of energy are markedly contrasted.

Thus, for instance, in the case of the moon and the earth, which revolve in stable equilibrium and in an orbit which is almost circular, a mean energetic distance of about 240,000 miles is maintained permanently by a mean linear speed of about 3100 feet per second. But, if the earth were devoid of rotation relatively to the sun, the friction of our oceanic tides would be tending steadily to slow down the moon. But as this happens the decreasing centrifugal force between moon and earth becomes no longer able to counterbalance the gravitational attraction between the two; and moon and earth tend to fall together. But as this occurs, space-energy is released, sufficient not only to make good the loss of energy to the tides, but more. The

equilibrium remains stable, and the abstraction of tidal energy has the apparently paradoxical effect of speeding up the moon.

As a matter of fact, the earth is revolving, relatively to the sun, in the same direction as the moon moves about the earth, and with a higher angular velocity. The result is that the frictional resistance of the tides, instead of tending to abstract energy from the moon for the speeding up of the earth, tends to slow down the earth's rotation, with the transfer of energy to the moon. In this case the moon is simultaneously removed from the earth and retarded in tangential linear velocity.

Either case illustrates the point, viz: that, whereas in radial motion the energy increases directly with the linear velocity, in tangential motion it increases inversely therewith. The mere alteration in direction from radial to tangential, which has been accomplished by gravitational reaction with the second mass-portion, has constituted a *reversal of algebraic sign of the energy*—algebraic signs, in energetics, being significant merely of whether the energy be going into or coming out of a given system, or of departure on one side or the other from the mean energetic condition. Although it is universally taught, in the engineering schools, that energy always enters matter as its velocity increases, and *vice versa,* it now appears that this is true of only one-half the energy of the universe—the radial, or perceptible, half. With the other half—the tangential, or latent, or imperceptible, half—velocities increase as energy departs from matter, and *vice versa.*

This innate tendency of all vibratory forms of energy periodically to alter the algebraic sign, with every transformation from kinetic to potential, or from radial to tangential, or the reverse —as visible first in the familiar pendulum, and now again in the action of any free mass-pair—is a fundamental characteristic of energy which is of the utmost importance. Nature knows nothing of smooth, continuous progress. All goes by pendulum-swings, reversals and transformations, as a ship tacks in beating against the wind.

It is in such terms as these that the energy-fund of the pair must be thought of, as consisting of tangential energy plus radial energy. The amount of *tangential* energy within the pair determines the mean energetic distance D, and the tangential component of the mean energetic velocity U. It remains perma-

nently fixed in form and quantity, so long as energy is neither added nor abstracted from without. The amount of *radial* energy within the pair determines the eccentricity of orbit and the radial component of the mean energetic velocity. It vibrates at each revolution from kinetic to potential form and back.

The Radial Energy-fund. The mathematical expression for the fund of radial energy within the pair may be had from Equation 7. By algebraic transformations which need not be reproduced here, it may be stated in terms of either the mean energetic or the periastron condition. It may also be expressed in terms of either space or velocity, for the energy alternately takes either form. It will also be of convenience for later purposes if the kinetic expression be stated in three different ways. Thus,

$$\text{Radial Energy} = E_r = \overset{\text{(potential)}}{2\,c\,M_1M_2\,\frac{e}{D}} = \overset{\text{(kinetic)}}{2\,M_1M_2\,\frac{U^2}{M_1+M_2}\cdot\frac{e}{1+e^2}} =$$

$$= \overset{\text{(kinetic)}}{2\,M_1M_2\,\frac{U^2\cos^2\alpha}{M_1+M_2}\,\frac{1}{e}} = \overset{\text{(kinetic)}}{2\,e\,M_1M_2\frac{U^2\sin^2\alpha}{M_1+M_2}}. \qquad (22)$$

Of these three kinetic expressions the first and simplest will be the most commonly used. Of the others, the last two will sometimes be convenient because, in the mean energetic condition, $U\cos\alpha$ is the radial, and $U\sin\alpha$ the tangential, component of the velocity U.

Stated in terms of the extreme energetic condition at periastron, S_0 being the periastron or minimum distance of separation and V the periastron or maximum velocity of motion, Equation 22 becomes

$$E_r = 2\,c\,M_1M_2\,\frac{e}{(1+e)}\cdot\frac{1}{S_0} = 2\,M_1M_2\,\frac{V^2}{M_1+M_2}\,\frac{e}{(1+e)^2} \qquad (23)$$

(It is to be noticed, in order to avoid confusion of thought, that at periastron, although the motion is there purely tangential in direction, radial energy nevertheless is present; because the tangential velocity is there so great that the radial forces are unbalanced. The radial energy takes this tangential form for an instant only.)

Of all the expressions for the radial energy of a pair, that

given in Equation 2, in potential form, is the first to get well in mind. Equation 2 reads:

$$E_p = c\ M_1M_2\ (\frac{1}{S_o} - \frac{1}{S}). \tag{2}$$

As the greater distance of separation S becomes very great, its influence upon the value of E_p becomes very slight; until, at the limit, when S has become of celestial dimensions between bodies of earthly magnitude, its value may be neglected. Equation 2 then becomes proportional to $\frac{1}{S_o}$ only. The same is true of the potential forms of Equations 22 and 23, wherein the space-factor appears only as $\frac{1}{D}$ or $\frac{1}{S_o}$. For this reason it seems convenient to assign to this reciprocal of radial space of separation the term *propinquity;* whereupon it may be said that, for all mass-pairs not already in unusual propinquity, their potential energy given out is proportional to their propinquity. And in any event it is true that differences in potential energy are proportional to differences in propinquity.

From this it becomes obvious that the amount of energy which must be abstracted from any mass-pair before they can be brought into coincidence, with $S_o = 0$, is infinite. Of course, no two bodies can ever be brought into coincidence. But since the obstacles to the feat lie only in the solid dimensions and the density of the two masses, which are variables to which no rule can be applied, it still remains true that the amount of energy in a pair which lies awaiting abstraction is indefinite in amount. *It is only the ability to abstract it which is limited.*

The Tangential Energy-fund. When attempt is made to give exact mathematical expression to the tangential fund of energy, trouble arises. In tangential motion no force is overcome by that motion, as is the case in radial energy. There is no transformation of energy. Indeed, there is not even any manifestation of energy. When careful thought is taken it appears as one of the fundamental principles in nature that *only radial energy can be perceived by the human senses,* or by any other external mass-system. A revolving pair, unless it be of such dimensions and velocity that its members can be perceived separately, as they alternately approach and depart from us, does not appear to us as a pair at all, but as two units.

Thus, the heavenly bodies are all of such dimensions that they can be perceived separately, and from them we get our first exact ideas concerning tangential energies. Similarly, such revolving pairs as fly-ball governors, fly-wheels, etc., usually revolve at a slow enough speed so that we can perceive their component parts; and so we learn to apply these exact ideas concerning tangential energies. But if the fly-wheel or the like revolves so rapidly, or becomes so minute, that its component parts are no longer distinguishable, then we can perceive its energy only when we come into contact with it as a whole. We then learn—if it does not contribute so much energy to us that our wits are at fault—that we have perceived only that portion of its tangential energy *which has ceased to be tangential and become radial.*

While this fact, like so many others which have been adduced to the present argument, is of comparatively little importance in engineering mechanics, it becomes of basic importance when the revolving pair, or system of pairs, reduces to the dimensions of a molecule or an atom, and its energy becomes known to us as heat or the like. When we touch a body and perceive that it is "hot" we are like some vast giant who might be too big to see our tiny human contrivances on the surface of the earth, yet who might perceive them by placing the tip of his finger upon a field in which were many rapidly revolving fly-wheels or buzz-saws. He could not see that each fly-wheel or buzz-saw was composed of many parts, balanced in their motion in an equilibrium which gives them the appearance of unity; yet to our senses it would be an obvious truth.

It is therefore not so surprising that, since we have no power to perceive tangential energy, we have no means for expressing it mathematically. For it is a fact that no exact statement can be made as to the tangential energy-fund existent within any pair. Tangential energy is there, surely enough; for it can come out, by becoming radial in form, and can overcome resistance. But there is neither any definite idea nor any exact equation for the total amount which can thus come out. We know, from Equation 21, that the amount thus available is one-half the radial space-energy available from any given same change of radial separation. But as the latter has already been shown to be, so far as exact mathematical limitations appear, infinite

in amount, the only statement which this leads us to is that the tangential energy-fund of any mass-pair is one-half of infinity; which is, of course, meaningless. It still leaves us forced to confess that for the tangential fund we have no exact expression.

Our knowledge in that direction is much like that, for instance, of a man who owned excellent thermometers, barometers, etc., all of which had slipped their scales. He would know exactly what a degree of alteration of temperature was like, or an inch of barometric pressure. He could report usefully to his neighbors, from day to day, upon the changes in the weather. But he could never tell them exactly how hot or cold it was, nor how far the conditions were from any absolute zero. So as to tangential energy, we can measure exactly, in joules, foot-pounds or what you please, any stated change in tangential energy, from known change of radius of purely tangential motion; but beyond that we cannot go. We cannot determine any absolute zero or maximum from which to make our ideas exact.*

In thinking of the energy-fund of any mass-pair or system, therfore, all idea of reducing the thing to any exact statement, founded upon an absolute zero, must be abandoned from the start. The only base or zero which has become visible in the preceding discussion—which has been just as exact as it has been possible to make it, short of dealing with infinitesimal masses—has been the *mean energetic condition.* It is only in this mean energetic condition that the tangential energy is directly visible. It is the amount of tangential motion on hand which determines the mean energetic distance D, or the space occupied by the pair.

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*In addition to the above it must be noted, before leaving this contrast between radial and tangential energies, that the radial energy alone may be considered the exclusive property of the mass-pair itself. It has already been noted that between a pair of absolutely single bodies only radial separations and velocities may be measured. For tangential motions the presence of at least three mass-portions is necessary. In nature this is tantamount to saying that no body is truly a single body. Every natural body possesses dimensions; and if so, tangential motion of revolution can be perceived by comparing its various portions. It is thus that we perceive the motion of the moon around the earth; as different portions of the earth swing up or down relatively to the moon, we say that the moon rises or sets. But to those telescopes which may be situated upon planets so distant that the earth appears as a point of light, the revolution of the moon can become perceptible only by a comparison with distant and apparently fixed stars; which amounts to bringing in a third mass very much larger than any of those yet mentioned.

On either side of this mean energetic condition the radial energy of every energetic pair vibrates as does a pendulum about its supports. But, in the case of the free energy-system, this support, instead of being fixed and definite, is floating in mid-space. It cannot be attached to or measured from any basis which has yet been devised. The most which can be done is to compare it with other mean energetic systems, each of which is equally homeless. As the sun serves as a central basis in reference to which the motion of the planets may be conveniently measured, so the mean energetic condintion serves as a central basis or zero-point from which, in either direction, the radial energy of the system may be conveniently measured. But both the sun and the mean energetic condition float indeterminately in space, without any possible reference to any absolute base or zero.

Indeed, the prime lesson sought to be imparted by this paper is that there is not anywhere in energetics, in any department where exact knowledge has yet penetrated, any reason for ever believing in the existence of an absolute zero for anything. So far as we now know, there is no absolute zero for either energy, velocity, space, force, volume, temperature or entropy; nor, so far as the writer is aware, for the similar factors of those other forms of energy with which he is less familiar, such as chemical, electrical, etc. All that we have ever perceived in any of them is a vibration on either side of some central mean value, which itself, in turn, cannot be regarded as fixed.

While this topic must receive further analysis, in later papers of the series, before it can be understood, it should be an attribute of energy familiar to every student, as its basic characteristic, that energetic potentialities consist as much in departures upon one side of the central mean as the other. For instance, in thinking of the energy embodied in mundane gravitation, such as water-power, it should be remembered that very low conditions of matter embody energy, as well as very high ones. When one wishes to purchase water-power it is usual to buy some basin which nature keeps filled with water at an appreciable elevation above the sea-level—which latter is in this case the mean energetic level. But if the valley of the Salton Sea, in California, or the Caspian or Saharan basins, or any other similar depressions below the sea-level, which nature would keep emptied

of water by evaporation, happened to be conveniently near the sea and on the market, they would constitute invaluable water-powers. With their help the waters of the ocean, which are commonly regarded as having reached the absolute zero of hydraulic head, would become gigantic sources of hydraulic energy.

Similarly with heat-engines, it is only by chance that it happens to be the most convenient thing, when one wishes power, to buy coal, fit to give out some 15,000 B.t.u. per pound at a temperature some hundreds of degrees above the mean thermal level for the surface of the earth. On the other hand, if a mine should be discovered from which could be procured a durable solid which would bear transportation, and which would *absorb,* instead of develop, some 15,000 B.t.u. per pound at a temperature even two hundred degrees below that thermal mean (whereas ice will absorb only about 140 B.t.u. per pound at a temperature of 32° Fahr.), the owner could sell it in unlimited quantities for heat-engine purposes, for chilling the condensers so low that ordinary sun-heat, even in winter, would suffice for the motive heat. If this novel substance happened to be more convenient than coal for any reason, all our better heat-engines would soon be designed in every line with a view to its use, just as they now are for the use of coal-made steam, or gas, or oil.

Nor is this mere phantasy. Since the temperature of our condensers is just as far below that of our steam-boilers as that of the boilers is above the condensers, our steam-engines must be regarded as evincing the availability of cold for work-performance as much as that of heat for the same purpose. If any steam-engine owner does not believe that he is running a cold-engine, let him allow his condenser to warm up. He will be in exactly the same trouble as if he allows his boiler to cool off. And as a matter of fact it is quite as likely that there exist somewhere in the universe deposits of substance which embody chemically the chill of interstellar space as that here on earth are deposits of coal which embody chemically the incandescence of intrasolar mass—though they are not likely to be convenient solids, like coal.

The conclusions which have been sought in the foregoing argument may be summarized briefly as follows:

1. The energy-fund of any mass-pair is made up of two contrasted sorts, viz: the radial and the tangential.

2. Of these, only the radial fund is capable of exact mathematical expression; or, so far as the argument has yet proceeded, of manifestation to any external mass-system.

3. The variations of either fund occur, not toward or away from any known absolute zero of any condition, but on either side of a mean energetic condition; and this mean energetic condition itself cannot be located absolutely, but only relatively to other mean energetic conditions.

4. The tangential energy-fund, itself unmeasurable, acts as mean central base for the radial fund, on either side of which the latter vibrates. For the tangential fund the mean central base as yet remains undefined; although it is obvious, from the generally stable equilibrium of energetic phenomena, that such a stable central condition must exist.

5. Of the total energy-fund of any mass-pair, the radial energy constitutes the perceptible, the measurable and the transmissible portion. It is the medium of communication between the pair and the outside universe. It may properly be styled the sensible energy. In its potential form it is distinguishable as either an unusual degree of spacial separation, or as an unusual lack of force (as at the apastron of the illustrative elliptic orbit). In its kinetic form it is distinguishable as either an unusual degree of unbalanced force, or as an unusual lack of spacial separation or unusual concentration of mass (as at periastron of the illustrative ellipse). These statements will be found to apply broadly to the thermal and mechanical energies of all forms of matter, as well as to the elementary illustration.

6. Of the total energy-fund of any mass-pair, the tangential energy is the imperceptible, the immeasurable and the non-transmissible portion. It is the elastic base upon which rests the radial energy. It is the means for the storage of perceptible energy received radially from without, and likewise the source from which is drawn the energy manifested radially to the outside world. It may properly be styled the latent or invisible energy. It is not to be perceived directly at all; and indirectly it is to be distinguished solely by its ability to turn into or absorb radial energy.

7. All energetic measurements must be made from, and all

concepts based upon, not any absolute zero of anything, which exists as a fixed support, but from a central, mean-energetic condition, which itself floats unsupported in mid-space, like our sun in the heavens. While the position of this mean-energetic condition must itself be subject to natural law, yet it is controlled by forces and phenomena too large to be taken into consideration in any concrete case.

8. Every mass-pair embodies some radial energy and some fund of tangential energy. This fact has not yet been developed in the argument, but it should be stated here, in company with the preceding seven principles. When the eccentricity of the mutual orbit of the pair is great, the radial fund is large in proportion to the tangential. When the eccentricity is small, the radial fund is small. But as the eccentricity can never be conceived as becoming either zero or infinity, there must always be some finite fund of radial energy. And since the pair can never be conceived as united into complete coincidence, but must always occupy some space, there must always be some fund of tangential energy. This idea will be developed later.

CHAPTER IV.

THE TWO FACTORS OR DIMENSIONS OF ENERGY.

In all of the mathematical expressions for energy which have been developed in the preceding chapters there have everywhere appeared two factors. One of the factors is the product M_1M_2 of the separate masses involved. The other is some function of either the space or the motion involved in their separation into duality. It is obvious that these two factors possess a distinct and contrasted significance.

The first of these factors, M_1M_2, is a measure of the extent to which duality exists, in which may be embodied the space or motion relationships which are measured by the other factor. It is a measure of the extent of "mass-pairing," as we shall call it for convenience, which is present. It has therefore been named by the writer the "extent" of energy, or the "extensity," present.

The second factor gives the degree of space or motion embodied within the mass-pair; and as it appears to human senses as the feature of energy which gives evidence of the degree of concentration of energy, by the sharpness of sensation or other effect which the energy may produce, it has been called by the writer the "intensity" of energy present. Of the two factors this latter is the more familiar to students of energetic phenomena, and will therefore be considered first.

The Intensity of Energy. If the fundamental expressions for potential and kinetic energy be divided by the factor M_1M_2, so as to derive mathematical expressions for the intensities of these two energies, there results, for the potential intensity, from Equations 2 and 22,

$$I_p = c\left(\frac{1}{S_o} - \frac{1}{S}\right) = \frac{c}{SS_o}(S - S_o) = 2\,c\frac{e}{D} \qquad (24)$$

and for the kinetic intensity, from Equations 5 and 22,

$$I_k = \frac{1}{2}\cdot\frac{V^2 - V_o^2}{M_1 + M_2} = 2\,\frac{U^2}{M_1 + M_2}\cdot\frac{e}{1 + e^2} \qquad (25)$$

From these equations the intensity of energy appears broadly as

a function of (1) for *potential* intensity a difference in "*propinquitics*," or reciprocals of spacial separation; and (2) for kinetic intensity a similar difference in *ratio of velocities-squared to aggregate mass involved.*

In kinetic mechanical engineering, it is the prime characteristic of the intensity of energy that it controls the direction of energy-transformations. It is a fact familiar to engineers that it is the body possessing the higher velocity, even if the smaller mass, which overtakes, collides with and gives its energy to the more slowly moving masses. Our entire experience with hammers, projectiles, and railway-collisions is but experience with the intensity of kinetic energy, and its promotion of energy transformation.

But now it appears that, whereas our present sub-conscious idea of kinetic intensity attaches itself merely to velocity, the exact affair is in reality the ratio of velocity-squared to aggregate mass. Like all other energetic phenomena, its manifestation here upon the earth's surface, with the earth for one of the participating masses in every pair, has been disguised by the fact that here the aggregate mass, viz: projectile plus earth, remains virtually constant, and so has been dropped from consideration of the variables. But when the bodies concerned are of molecular magnitudes, with nothing known as to comparative masses of the members of any pair, it is obvious that the denominator of the ratio mav become as active a factor as the numerator, in the variation of intensity.

As to the potential intensity of energy, that is also a familiar fact in every-day engineering; but not quite in the form presented here.

In the first place, potential intensity, or intensity of energy due to an unusual departure of spacial separation from the average, may assume either of two forms, viz: unusual separation, on the one hand, or unusual lack of separation on the other. That is to say, potential energy may be observed as great either because S is very much greater than the mean energetic distance D, or because S_0 is very much smaller.

All of the cases in engineering which are commonly recognized as potential mechanical energy, such as suspended weights, mill-pond water, or projectiles at the summit of their trajectories, belong in the former class. They are in reality following the

apastrons of extremely eccentric orbits, which pass closely about the center of the earth—which orbits, of course, can never be completed because of the solid obstruction of the earth's body. Their mean energetic distances are a few miles at most, perhaps a few feet only, from the earth's center; whereas their separations from that center while we are using them is some four thousand miles, the earth's radius. Moreover, they possess very little tangential motion parallel with the earth's surface. Therefore, their gravitational force is almost entirely unbalanced. We consider $\frac{c}{SS_0}$, in Equation 24, to be a constant, and S_0 in the parenthesis to be negligible. The distance of separation S—or *h,* for *height* or *head,* as it is commonly called—remains alone, ready for us to consume of it what small fraction we may.

But we also know potential intensity, in engineering, as an unusual lack of separation in mass. As we compress an elastic gas, or a steel spring, for instance, into a volume less than its condition of mean energetic equilibrium, there is developed a latent potentiality for energy which manifests itself as an unusual intensity of *force.* This force does not appear to the average engineer, it is true, as an integration of the many unbalanced centrifugal forces of a multitude of tiny orbits at periastron. But there is no other unbalanced force in the elements of mechanics which so fits the conditions as to be acceptable as an explanation of it.

Force, it is also true, is a function not only of $\frac{1}{S}$ (or $\frac{1}{\text{volume}}$, as we should state it for gases), but also of the masses involved. Therefore it is not a pure manifestation of intensity. But when the mass-factor happens to be constant, as is commonly true in engineering mechanics and as may sometimes be true even in molecular mechanics, it becomes a true measure of intensity.

In this case, also, the characteristic of intensity as the determinator of the direction of energy-transformation appears. It is always the greater force which overcomes and contributes energy to the smaller force. It is always the smaller force which receives and stores it. That is why our fundamental concept of energy covers the action of all the unbalanced force present, acting through an infinitesimal distance, rather than all the distance covered, acted upon by an infinitesimal element of the force.

That is to say, the mathematical equation for this fundamental concept of an infinitesimal element of energy is $\mathit{Force} \times d(\mathit{Space})$, and not $\mathit{Space} \times d(\mathit{Force})$.

Therefore, while the mathematical equations are the only true guides, yet the general concept of potential intensity may be based upon either *force* or *space,* force being understood to be synonymous with *lack* of space, or reciprocal of space. When the intensity is in the form of unusual space between the mass-portions, the unbalanced force is centripetal, gravitational or concentrative, and is comparatively slight. When the intensity is in the form of unusual lack of space, the unbalanced force is centrifugal, or disgregative, and is very great. The first offers us much distance traversible with little force; the latter makes available a greater force operative through a smaller distance. The first typifies the action of suspended weights; the latter that of compressed elastic matter.

Thus, either unusual attenuation or unusual condensation of matter constitutes potential energy. Unfortunately, at present we quite lack suitable names for these two contrasted types of potential energy. If we were to coin words for them, *disgregic* and *congregic* energies would be the natural names. But this is merely a suggestion.

In the above statements all words of degree, such as great or little, are used only in a comparative sense, for any given mass-system. For in natural mechanics all human standards of dimension disappear. The mass-action of solar systems, baseballs and molecules is now believed to be the same, in principle. Any such a system may be taken as a standard of dimension; but the ideas of great or small applied from these different bases must be understood as fitting widely different experiences.

The Extensity of Energy, or Degree of Mass-pairing. The extensity-factor of energy M_1M_2 appears always as a product of two separate masses, rather than as their sum. This is so because energy can exist only *between* masses, and not throughout mass. That is to say, the symbols M_1 and M_2 signify that within the first body there reside M_1 units of mass, while in the other body reside M_2 units. Any single mass-unit in M_1 would then form, with the several single units in the other body, and across the gap between the two bodies, M_2 *unit mass-pairs.* The entire community of mass-units in M_1 would therefore form

$M_1 \times M_2$ unit mass-pairs across the gap separating the two bodies.

Remembering that energy can exist only in the gap between separate mass-portions, and never in the mass itself, it becomes clear that the factor M_1M_2 is the *extent of mass-pairing* which is energetically active across this gap. It is clear, too, that in any energetic system the total extent of mass-pairing present, or Σ MM, in which to embody intensities of relationship, is just as much a factor, and may be just as variable a factor, in the energy as is the intensity of relative space or motion itself. It is because we have so long been accustomed, in our engineering, to splitting off from the earth the mass of a railroad-train or a cannon-ball and treating only of its energy relatively to the earth—in which case the mass-pairing factor remains constant so long as the separation between earth and machine, or the energy itself, lasts—that it has been forgotten that the mass-pairing factor is itself as likely to experience variation as is the intensity-factor of relative space or motion. And in molecular mechanics it is perhaps even more likely.

This variation in extent of mass-pairing may take place in either of two ways. First, the mass of either party to the energy may increase or diminish; in which case $M_1 + M_2$, or Σ M, varies with Σ MM. This, according to the ordinary teachings of mechanics, is what always occurs. We are taught, in fact, that we can increase the energy between cannon-ball and cannon (for a fixed muzzle-velocity) only by increasing the mass of both cannon and projectile.

But there is a second method of increasing the massivity of energy present, and that is by increasing the *number* of cannon and projectiles. At first glance, this seems to be the same as before, viz: energy increasing in proportion with mass. The difference appears when it is remembered that the earth is a party to almost all engineering energies, and that their true nature comes out only when matters are expanded to a scale commensurate with the dimensions of the earth. The earth weighs about $6 \times \overline{10}^{21}$ long tons. A cannon and projectile which together weighed this amount, and yet possessed only the muzzle-velocity of standard cannon, would be a very mild affair, as planetary energies go. But compare with this $6 \times \overline{10}^{21}$ long tons of standard cannons and projectiles, all going at once, and

each of the projectiles equipped with energy relatively to the entire remaining mass—and some concept may be had of the energetic possibilities of mass-subdivision! In comparing these two cases, the Σ M, or aggregate mass, of the two are the same; but in the latter case the Σ MM, or extent of mass-pairing, is very much the greater.

The distinction involved here is hard to bring out clearly, for man has had no experience with mass-systems consisting of an earth's-weight of cannon, all going off at once. Yet this is the only true mechanical simile, in comparison with the placid old moon-earth system, to a white-hot cannon in comparison with the same cannon cold and solid, discharging a single shot. It is because human experience covers no gradations between these two extremes, of matter acting as a solid unit on the one hand or as an innumerable multitude of separate solid units on the other, that we have always regarded "work" as one thing and "heat" as a very different thing—that we have arranged all our formulae for mechanical energy with the mass-factor appearing as M instead of as M^2, neglecting entirely the wide range of variation of which the factor Σ MM is capable while Σ M remains constant.

Yet this distinction is all-important, as lying at the heart of the comprehension of any of the more obscure forms of energy, such as heat. In order to grasp it, it must be remembered how necessary has been the common habit of speaking, even in celestial mechanics, of the bodies involved as if they were solid homogeneous units. In discussing the energy embodied between moon and earth, for instance, it has been customary to consider both moon and earth as perfectly solid, unit spheres, each acting as if concentrated at its own center. Yet this is very far from the truth. Each of these "units" is in reality extensively and minutely subdivided into interacting energetic mass-pairs. We who inhabit the earth know full well of that planet, if not also of the moon, that every energy which forms the subject of a human science, in mechanics, hydraulics, meteorology, chemistry, biology and electricity, stands in evidence of minute subdivision of mass. The earth, instead of being a homogeneous unit, is in reality subdivided into a vast and most intricately organized host of parts, of differing densities and other characteristics; and between these divided, distinct and independent particles, of

rock, water, air and the various chemical elements, exist the most diverse funds of energy, both potential and kinetic. Merely to mention the unquestionably mechanical ones, there are the energies of the winds, waves, tides and waterfalls, all the ponderous machines of human construction, and the innumerable host of moving arms, legs, wings, fins and claws besides. There is not only energy in the abysmal gap between moon and earth, but there is energy in each of the myriad of tiny crevices in each. Wherever exists subdivision there exists energy.

Indeed, all that is meant, in speaking of the moon and earth or any other body as a "solid" unit of mass, is that *for the particular purposes of this particular sort of energy* the mass-portions concerned may be considered as solid units; that is, acting as if their mass were concentrated at a single point. So long as this special assumption holds good, ΣMM must be regarded as a constant, as well as ΣM, and the amount of this particular sort of energy is proportional to the first power of the mass involved. But, considering all sorts of energy together, we know of no instance in which this special assumption holds true broadly. Human experience has never yet encountered a homogeneous or true solid. Like the geometric point or line, the solid state of matter is merely a convenient figment of the human brain.

It has been altogether natural, and even necessary, in the study of applied mechanics, to consider mass thus, as lumped into portions which were solid units, acting perfectly in unison. But for a mechanical concept of heat such an idea is fatal; and even for the purposes of applied mechanics it seems to the writer to have been overdone. For it is essential, for the comprehension of any of the several other energies which are daily growing in importance, to have constantly in the mind's eye a picture of mass, not as a homogeneous unit, or solid, but as a heterogeneous system, subdivisible again and again, into smaller and smaller portions, just as far as human perception is able to penetrate—into molecules, atoms, ions and what not. The history of science justifies no other view. Each new stage of scientific progress has revealed some further refinement of subdivision of matter which had previously escaped the more clumsy perceptions of cruder times.

The difference to the comprehension of the general nature of

energy lies in the fact that if the homogeneous dogma be true, the purely mechanical energy which may be embodied in any given mass is limited by the velocity or distance relatively to some fixed base, such as the sun, which may be imparted to it; in short, to a variation in the intensity-factor of its energy. Thus, in comparing the energetic possibilities of different masses, these must vary directly as the mass. But if the heterogeneous dogma, based upon the equations given in these papers, be properly understood, then mass appears as capable of embodying within itself any amount of energy whatever, not only by variations in its intensity of energy, but also by variations in its extensity of subdivision, or comminution.

This can perhaps be made clearer by an illustration.

Let us consider a form of energy which we may call *military* or *combative* energy. Suppose there exists an army of twenty thousand men. If one of these men should desert his duty, the power of the remaining nineteen thousand and odd arrests him —perhaps only after some desperately violent "intensity" of resistance, the utmost combative energy that any one man might arouse in the face of superior numbers. This, in the particular illustration chosen, is the minimum possible degree of subdivision of the army's mass into combatively opposed portions. The *extensity* of combative energy present is given by multiplying one by 19,999; while the *intensity* of combative energy depends upon the strength and spirit of the individual thus split off from the corps.

The portion of the army thus split off, in disobedience, of course might be greater than a single man. The number in revolt might grow most gradually, man by man, until it had become full half the army. If we suppose, for simplicity, that each man's intensity of combative ability were equal to that of his fellows, then the energy of combat would grow, as the revolt spread, proportionately to the product of the numbers engaged upon the two sides. And this would reach a maximum, at $10{,}000 \times 10{,}000 = 100{,}000{,}000$, when the opposed parties had become equal.

But this is not the maximum extensity of combative energy of which the army is capable. It is merely the maximum attainable when only a single subdivision, into only two portions, occurs. Suppose, however, that by the time the revolt had

absorbed one-half of the army it became civil war, one side adopting a red uniform and the other a blue. Suppose, then, that the spirit of disaffection so spread that further subdivision took place within each faction. The red army splits into a "yellow" and a "gray," of five thousand men each, while the blue army splits similarly into a "green" and a "purple." Each of these new armies raises a banner of its own, and opposes any and all comers. What is now the possible number of individual personal conflicts? What is now the extensity of military or combative energy?

Between the four armies are possible six different battle-arrays—a yellow-green, a yellow-gray, a yellow-purple, a gray-green, a gray-purple and a green-purple. But the possible number of personal conflicts in each battle has now been reduced to only $5000 \times 5000 = 25,000,000$, or one-quarter as great as before. The aggregate number for all the armies, together, therefore, is but $6 \times 25,000,000 = 150,000,000$. This measure of the extensity of combative energy proves to be only fifty per cent. greater than when the army was divided into only two equal portions.

But, obviously, the extent to which the army may subdivide into equal portions, in mutual discord, is not limited to four such parts. The possibilities in this direction are not exhausted until each individual soldier has become a knight errant, under his own standard, and the field a proverbial Donnybrook Fair. The extensity of combative energy would then be much greater than 150,000,000, it is clear; and yet it is equally clear that it would not have grown in proportion to the number of subdivisions.

Also, it is to be noted before passing to mathematical exactness in the discussion, the degree of subdivision of the army is limited to the degrees specified, only because the discussion has been confined to *military* energy, a form in which the unit mass-factor is a single soldier-pair. That is to say, for this special purpose, it has been assumed that each soldier is a solid, homogeneous, indivisible unit, containing no internal subdivisions and energies. This, of course, is not true in speaking of energies in general; for each soldier is a most intricate conglomeration of separate organs, muscles, glands, bones, cells, etc., and is capable of embodying much physiological energy even in times of peace, when the army remains a solid unit. Only, for the particular

purposes of combative energy the unit mass-pair is a couple of soldiers, completely equipped; just as for the particular purposes of heat-phenomena the unit mass-pair must be considered as two somethings called molecules, for chemical energy two somethings called atoms, for electrical energy two somethings called electrons, etc., etc.

If this general idea be reduced to a mathematical basis, it will appear that the extent of mass-pairing, or extensity of energy, X, of which any aggregate mass M is capable, grows with the number of equal parts *n* into which the aggregation is subdivided, according to the equation

$$X = \tfrac{1}{2} M^2 \left(1 - \frac{1}{n}\right) \qquad (26)$$

From this, if $n=1$, $X=0$. If $n=2$, $X=\frac{1}{4}M^2$. If $n=4$, $X=\frac{3}{8}M^2$. If $n=100$, $X=0.495M^2$. And as *n* grows indefinitely larger the value of X approaches more and more nearly and slowly to $\frac{1}{2}M^2$, which value it can never reach.

On the other hand, if *n* becomes less than two the value of X becomes a fraction of $\frac{1}{4}M^2$. Such would be the case when the aggregate mass was divided into two portions, but not equal portions, the value of *n* becoming one plus a smaller and smaller fraction as the one portion became a smaller and smaller fraction of the whole. Such is the case, for instance, in the energetics of engineering, if the mass of the earth be considered as the unit of measurement. As we split off from the earth portions which we manufacture into cannon-balls, railroad-trains, etc., we are in reality giving to *n* of Equation 26 the value of one plus a very small fraction. The energy becomes zero, because its mass-factor has become zero, only when the value of *n* becomes unity, signifying that the earth is a homogeneous solid, possessing no dissevered parts like cannon-balls.

If *n* becomes less than unity, the value of X becomes negative. Such would be the case when the entire mass in question is but a part of a single member of some much larger mass-pair. As the value of *n*, which must always be a positive quantity, approaches zero the value of X approaches negative infinity.

The application of this formula to the understanding of energetic action will be made in later papers.

Hitherto the principles of mechanics have usually been discussed in the text-books only in terms of constant mass-quantities. Indeed, there is no college-treatise yet come to the writer's attention which gives to the student even an inkling of the fact that the mass-factor in energy may be—to say nothing of the fact that it always is—just as variable and active a factor as the space or motion factor. For the purposes of applied mechanics this is quite sufficient; but so soon as the general field of energetics is entered—as it must be by every modern student in even the specialized branches of engineering, wherein are constantly met transformations between heat, work, chemical and electrical energies—the method promptly becomes disqualified. The student should be taught, as soon as any general energetic concepts whatever are presented, which is usually in the study of thermodynamics, in the junior year, that the mass-factor of energy is just as frequently and widely a variable as is any other; and that the mass-factor is not merely mass, but may be anything between the first and second powers of mass.

To accomplish this, between the course in the elements of applied mechanics (usually taught as a part of the general course in physics) and the course in thermodynamics, should be inserted a separate course upon the *true* elements of mechanics and mechanical energetics, as outlined above. The approximate formulae of applied mechanics the student is to continue to use in his engineering problems. The true formulae are to furnish him with his concepts of truly natural mechanical action, with which he is to interpret the obscure phenomena of thermal, chemical and electrical interactions.

In this, the basic fact now needing especial emphasis is that when energy is imparted to mass it may find embodiment therein in either of two general ways; and in nature these two ways occur, not only with equal frequency, but always in combination. First, it may increase the relative velocity of motion, or space of separation, of mass-portions which are already in separate existence; that is, it may increase the *intensity* of energy. Secondly, it may subdivide mass which was previously unified into newly separated mass-pairs, into each of which is injected a first measure of relative separation or relative motion, or both; that is, it may increase the *extensity* of energy.

It has been too commonly taught that only when we raise

weights or accelerate cannon-balls do we embody energy in mass, and too seldom taught that when we crush rock or grind cement we do the same. Only, when the cement is ground or the rock crushed the resultant particles do not embody, in their disgregation, an energy which we can utilize again. So that we say that it is "gone"; although all of it which has not become heat lies there right before our eyes. And even as to the heat, that appears, upon inspection, to be merely a finer degree of comminution and disgregation than that embodied in the visible particles. For not only will the same processes which grind rock into powder also grind water into steam-heat, but the latter is, almost equally with the former, unavailable for further energetic use.

The student has been taught from the start, in the doctrine of the Conservation of Mass, that mass can be neither created nor destroyed. But has he been taught with equal care that there is no known limit to its aggregation or subdivision? Or that these processes are going on at all times, in nature? Or that energy is as much involved in these processes as it is in the accumulation or dissipation of motion or space?

Nor can any limit be placed to the value to the student of an exact concept of this great natural fact, in after life. It may be only suggested here how wide may be its useful application. It is not alone in thermal, chemical and kindred phenomena that subdivision, specialization and organization of subject-matter into coöperation are of prime importance for the effectiveness of energy. Before the student has reached college he has learned, upon the play-ground, to substitute skill for bull-strength; he has learned that athletic energy must be subdivided and organized into team-work before games can be won, or even played, which are worth while. As college-student and embryo manufacturer he learns that shop-organization and office-specialization are prime factors of success in business. If he joins the militia he learns the importance, in military or combative energy, of the solidification of bodies of men for the resistance of shock; but of their subdivision and specialization to a high degree, until virtually each individual does a different thing, in order to embody in them the highest degree of combative effectiveness. Should he study law he learns from the most eminent jurists that the whole business of the law is to define and determine the *relationships* which are to prevail, as natural, between the

individuals of our ever more numerous, more diversely intricate and more energetic race; and natural relationships are just what these pages are to define and make familiar, by study of them as they exist between the simplest possible individual elements.

It is further the writers' prediction that before to-day's student has reached middle-age he will have learned, from forcible and costly experience in helping to make his country's history, that what that country has needed most, for decades, is a similarly accurate concept of the natural relationships between man and man, in the day's work. He will have learned that what it has long urgently needed, and soon can exist no longer without, is an economic organization, throughout its every economic activity, similar to that now prevailing within each factory. Within each factory we now have superlative discipline; but *between* our factories prevails superlative anarchy and civil strife. We are relying too much at present, for prospective cure of our economic troubles, upon greater *intensity* of individual effort. We are too unconscious of the possibilities of greater *extensity* of coördinated energy. We scarcely know yet what the words mean.

Whether a student is to become an engineer, a teacher, a business-man, a lawyer, a preacher or a statesman, his life-work should be founded upon a clear understanding of mechanical energetics. In each of those fields, if his education is to aid his life-work, he must have at his fingers' ends the fact that energy can be accumulated by accumulating mere mass; but that then it partakes of the nature of solid mass. Solidity, rigidity, inflexibility, hardness are its characteristics; impact and friction are its results. But that if, with the accumulation of mass—or even without it—go subdivision, specialization and coördination into increasing extensity of mass-pairing in energetic interaction, then fluidity, flexibility and efficiency in work-performance become its characteristics. In addition, the *amount* of energy which can then be embodied in any given mass is extended indefinitely, absolutely without limit foreshadowed by those mathematical formulae which are yet available. This is as true of masses of men as of masses of matter.

CHAPTER V.

THE EXTREME OR CRITICAL ENERGETIC CONDITIONS.

In considering the mechanical elements of the original freely moving mass-pair, early in this series of papers, it was established that its energy vibrated, during its progress along a conic-section orbit, on either side of a *mean energetic condition.* This mean energetic condition was identified as occurring at the extremities of the latus rectum of the conic-section orbit, twice in each revolution.

The aspects of the different possible forms of orbit which were listed in Chapter II, viewed in reference to this mean energetic condition, may be stated as follows:

1. If the orbit be *circular* all points of the orbit embody the mean energetic condition, and no energy-transformation whatever occurs. The pair is perceptible only as a unit. Its orbital motion is evident to man only in its occupancy of space and its resistance to compression; that is, by its permanence and indestructibility.

2. If the orbit be *elliptic* there are two mean energetic points in each revolution, through which energy-transformation occurs periodically, first in one direction and then in the other, like the swing of a pendulum; and this process continues indefinitely.

3. If the orbit be *parabolic,* while there are still two mean energetic points per revolution, yet there can be only one swing of the pendulum; and on its outward swing the energy-transformation tends toward a complete balance, with no residue. That is to say, as the bodies swing apart, converting kinetic energy into potential, the former is just almost, but not quite, absorbed in accomplishing the utmost separation which is imaginable for the pair. As the bodies separate very widely their velocity almost becomes zero; but it never quite does so, for the gravitational attraction will then also have become almost zero, so that there is little tendency to stop separating and reverse into a mutual approach. But this perfect balance of kinetic and potential energies may be regarded as so very unlikely as to be

non-existent in nature. Virtually, all orbits are either elliptic or hyperbolic in form and nature.

4. If the orbit be *hyperbolic* there is not merely only one swing of the pendulum and only two mean energetic positions, but there occurs permanent dissociation of the pair afterwards. The kinetic energy at periastron is so great that, even after separation to an infinite distance, there still remains a finite residue of velocity of separation, which can never be absorbed potentially by the pair.

5. If the orbit be a *straight line*—which is the limiting case of an hyperbola with infinite eccentricity—there is no discernible mean energetic condition, no appreciable perturbation of path by the pair's propinquity (which is zero), and no exact measure for the energy involved. All quantities have passed to either zeros or infinities, neither of which the mind of man can grasp. The situation is to be mentioned, not as a natural fact, but as a mathematical limiting condition, to which natural phenomena may never attain; and also to remind the student how unnatural is the straight-line path of motion, followed by matter only when under constant constraint.

Let this question be illustrated by means of a somewhat familiar mundane mechanism.

Imagine a cannon placed upon a platform elevated ten miles above the surface of the earth, aimed horizontally, and fired. Its projectile will start upon an orbit which is commonly described as a vertical inverted parabola. But this is true only upon the erroneous assumption that the earth is a flat solid of indefinite horizontal extent, its lines of gravitational attraction being parallel vertical ones, of equal intensity at all distances from the earth. But Newton himself proved that the gravitational effect of a sphere of appreciable dimensions followed his law exactly, even when distances very small in proportion to its diameter were considered, just as if the mass were concentrated at the sphere's center. The true definition of the projectile's path is therefore an ellipse, having its further focus coincident with the earth's center. This path, however, the projectile cannot prosecute far before it is interrupted by collision with the earth's solid surface.

Suppose, however, that muzzle-velocities might be increased indefinitely. The process would in reality be one of adding

energy to the pair tangentially, at apastron. The ellipse would then grow wider, the eccentricity less, and the mean energetic distance from the earth's center greater. When the muzzle-velocity had attained to some 26,000 feet per second the elliptic orbit would have become widened into a circle, clearing the earth's surface and its mountain-tops, and returning the projectile to the point of its start once each eighty-five minutes. If the cannon had been wheeled out of the way during the progress of the first circuit of the earth, and if the atmosphere were absent, the projectile would continue thus as a satellite of the earth, in circular orbit, indefinitely.*

Suppose now that the muzzle-velocity of the projectile might be still further increased. Its orbit will now have become elliptic again. But now the point of start is located, not at apastron, but at periastron of the orbit. The projectile, at apastron, is "over" the antipodes; and at each increase of muzzle-velocity its height above the antipodes increases. The eccentricity of orbit now increases again. The major axis of the elliptical orbit elongates, and the period of time elapsing between each two returns of the projectile to the platform increases, according to Kepler's Third Law, from the original eighty-five minutes by the three-halves power of the major axis.

Finally, when the muzzle-velocity should have reached and surpassed some 37,000 feet per second, the orbit would have become, first parabolic, and then hyperbolic; and the projectile would depart from the earth forever. Any higher muzzle-

*This velocity is derived from Equation 17. Taking the values for g as 32.18, for the radius of the orbit as 3,960 miles and the mass of the earth as something less than 42×10^{22}, it appears from Equations 7 and 17 that the velocities of equilibrium are as follows:—

	Ft. per sec.
Circular motion around the earth, just above its surface,	26,000
Vertical motion, directly "up" or away from the earth, sufficient to carry the projectile indefinitely into space—or contrariwise, the velocity of striking the earth after free fall from space directly toward the earth's center,...........................	37,000

The last figure gives the maximum velocity of relative motion which can exist between a mass-portion of the density and dimensions of our earth and a mass-portion considerably smaller, as the exclusive property of the pair, and which can continue indefinitely, without dissipation in either collision or dissociation. Should higher velocities than this be observed they must be the property not of the mass-pair between which they are observed by the eye, but of some much larger mass-system. Only such a larger mass-system could either generate or arrest such excessive velocities.

velocities than this would, of course, only accentuate the promptness of this dissociation.

For any given mass-pair—and virtually, for any given major mass, when the individual masses are widely dissimilar—this "critical" velocity at periastron, above which dissociation takes place, is a definite thing, for any given periastron distance S_0. For, as was noted in the preceding paper, the expression for the spacial intensity of energy of a pair is $c\left(\frac{1}{S_0} - \frac{1}{S}\right)$; and if, in this, S becomes very great indeed, the intensity takes the definite value $\frac{c}{S_0}$. Therefore, if two bodies fall together from any distance of separation whatever, however great, into a minimum separation of S_0, this is the utmost intensity of motion which they can develop. In the case of the earth this velocity is about 37,000 feet per second. Conversely, two bodies situated at the minimum distance of S_0 require only this intensity of motion to dissociate them permanently.

This degree of intensity, then, is the utmost which this particular pair is capable of embodying, unless conditions permit them to approach nearer than the distance S_0. Ordinarily, S_0 is limited by the finite dimensions of the bodies, when solid; but theoretically there is no limit to the diminution of S_0, and as it diminishes the intensity of energy increases very rapidly.

But at present it is not so important to discuss the possible variations of S_0 as it is to note that, if any pair be observed at periastron with an intensity of motion greater than the "critical" one, corresponding to $\frac{c}{S_0}$, implying a hyperbolic orbit and prompt dissociation, it signifies that the pair visualizes what it cannot, of itself, embody, viz: a fund of intensity greater than $\frac{c}{S_0}$. The surplus of energy over this quantity cannot possibly be the property of the pair itself. It is a manifestation by the pair of an energy-fund held by it in conjunction with some third external mass-portion, which third portion is itself not necessarily visible directly as a member of the system. But nothing but some such larger third mass-portion is capable of destroying the

surplus velocity, and none but it could have been capable of producing it.

This surplus, or third-mass, energy may be called, for the present, the "external" energy of the pair, in order to keep our argument in terms of a simple mass-pair as a base. This external energy the pair has borrowed and displayed as its own, so to speak, although it has no rightful ownership of it and must soon repay the loan.

The relation of such external obligations to its own internal assets of energy is visible directly in the value of the eccentricity of orbit e. For it is only when e is greater than unity that hyperbolic motion and dissociation can occur. The excess of e above unity, therefore, is a measure of the external or borrowed energy which is displayed by the pair.

In order to understand this situation completely, return must be made for a moment to the question of radial and tangential energies.

A value of e greater than unity implies an angle between mean energetic motion and the radius vector connecting the two bodies (see Fig. 4) smaller than 45°. In that case the radial component of motion would be greater than the tangential. Here, again, one must be careful with his mathematical expressions for energy. If it be assumed, too readily, that kinetic energies are absolutely proportional to the square of the velocity, then the ratio of radial to tangential kinetic energies, at the mean energetic point, must be e^2; for $e = \text{cotan}\ \alpha$, the ratio of radial to tangential velocity at this point.

But two objections exist to this reasoning. In the first place, no exact expression for the tangential energy exists at all. In the second place, if there were one it must be *negatively*, rather than directly, proportional to the velocity squared; for energy must be abstracted in order to increase the tangential velocity.

The way out of this puzzle is to note, from Equation 22,

$$\text{Radial Energy} = 2\ \text{c}\ \text{M}_1\text{M}_2 \frac{\text{e}}{\text{D}} = 2\ \text{e}\ \text{M}_1\text{M}_2 \frac{\text{U}^2\ \sin^2\alpha}{\text{M}_1+\text{M}_2} \qquad (22)$$

that, for any stated mean energetic conditions, such as distance D and tangential velocity U sin α, *the radial energy is directly proportional to the eccentricity of orbit e.* And since the mass-pairing factor M_1M_2 is always alike on both sides of this equa-

tion, bearing no influence upon the result, it would have been more explicit if this statement had been made in terms of intensities of energy, rather than of energy itself. Thus it may be considered proven that when the radial intensity—which has already been characterized as the medium of communication, or manifestation, between the pair and the external world of mass—assumes a proportion to the internal, or stay-at-home, intensity greater than equality, then dissociation and a cessation of the pair's visible existence must ensue promptly. It is with a molecule much as it is with a man: when he goes too energetically into foreign politics, to the overshadowing of his private business, his home breaks up.

The Lower Critical Intensity. It has already been suggested, however, that something else than dissociation may happen to interrupt the continuity of the pair's energetic existence in smooth, unbroken orbit, with energy manifestly conserved. Collision may occur, at or just before periastron.

For this to take place, the distance of separation at periastron must be less than the sum of the radii of the two bodies. The situation must therefore be investigated through the energetic equation in terms of periastron conditions, or Equation 23.

$$E_r = 2\,c\,M_1M_2\frac{e}{(1+e)}\frac{1}{S_o} = 2\,M_1M_2\frac{V^2}{M_1+M_2}\frac{e}{(1+e)^2}. \quad (23)$$

The last two terms of this equation readily reduce to

$$\frac{c}{S_o} = \frac{V^2}{M_1+M_2}\cdot\frac{1}{1+e} = I_c \quad (27)$$

which is the fundamental equation for the critical intensity, in which the first two terms define the limits of spacial concentration, on the one hand, or of embodiment of internal motion, on the other, which respectively constitute the criteria of energy-transformation.

If, in Equation 27, S_o be considered equal to the sum of the radii of the two bodies, the expression gives the limiting condition by which collision at periastron may be avoided.

The Critical Limits of Intensity. Upon these considerations may be founded the statement that the determining factor in the permanence of energy is its intensity. Within certain limits the intensity of a given pair may vary; with effect upon the external appearance of the energy, it is true, but without pre-

cipitating a transformation of the energy into some other apparently independent form. But as soon as these limits are surpassed, the energy alters its outward aspect so radically as to make it difficult to discern the true continuity of its existence. We are constrained to say that the energy has therein become "transformed," so that we give it another name than before. And as the understanding of the more obscure forms of energy is inextricably connected with energy-transformation, which gives them birth and death, it is clear that the entire science of energetics turns upon these critical limits of intensity as a car does upon two king-pins.

These limits, and their effects, may be stated briefly as follows:

1. If the kinetic, or outward radial, intensity exceed the limit implied by the condition $e = 1$, the pair will dissociate.

2. If the spacial, or inward radial, intensity exceed the limit implied by the condition $\frac{1}{S_0} = \frac{1}{R + r}$, wherein the R's are the radii of the two masses, the pair will collide. (For the intensity increases, it will be remembered, as S_0 grows smaller.)

Contrasting these two sorts of critical condition, in terms of the contrasted aspects of the two extremes of the simple orbit, the first is perceptible as a critical limit to the *spacial expansion* of the system, by too great velocity radially outward—in which phenomenon force plays little part. The second is perceptible as a critical limit to the *forceful compression* of the system, by too great a velocity radially inward—in which phenomenon space plays little part. It will develop later that the first form of critical condition is of interest in connection with the too great expansion of vapors and gases. The second is of interest in connection with the too forceful compression of solids. For either procedure will develop energy-transformation in bodies of matter.

After dissociation occurs, the further history of the pair is about as easily written as is that of the snakes in Ireland. Virtually speaking, there is no further history. The pair has ceased to exist, as a pair, though its two members continue an indestructible existence.

Yet in this facile statement, which is necessary in order to take the road step by step, lies in reality a great error against

which the student should be warned. In truth, however distantly (and apparently permanently) a mass-pair may be separated, by chance foreign influence, yet it is never completely sundered, and never can be. Though the distance of separation may become that which now lies between any object at hand and those scattered throughout the farther heavens, at distances beyond human comprehension, yet the mutual bond of attraction still exists, in the case of each pair, to a measurable degree. And it exists eternally. Though the number of centuries which must elapse, before chance again frees the pair for a fall into mutual propinquity and perceptible energetic reaction, may far surpass human understanding, and strain even our arithmetic to compass its expression, yet when the time does come the latent mutual affection will be just as potent for warmth of greeting as if it occurred to-day.

As to the further results of collision, on the other hand, that topic lies much nearer to the purpose of these papers than does dissociation. Indeed, its bearing is so vital that it will be postponed for a special chapter of its own. In the meantime, attention may be turned to those processes which may lead the energy of any mass-pair to exceed the critical limits of kinetic or spacial intensity, with the results just defined.

Collision and consolidation of the pair into unity may result from the abstraction of energy in either of two ways, viz:

1. If the energy be extracted in *radial* form (that is, between periastron and apastron, by a medium capable of absorbing only the radial component of motion), the eccentricity e decreases. Because the tangential component has remained unaffected, the mean energetic distance D will remain unaltered. The motion becomes more nearly circular, and thus lapses toward a peaceful consolidation; for it was explained in an earlier paper that true circularity of motion constituted an apparent unity.

2. If the energy be abstracted in *tangential* form (that is, at apastron, by a medium capable of absorbing only tangential energy), the eccentricity e is increased while the mean energetic distance D is decreased. From both reasons the periastron distance S_0 decreases. The motion becomes more nearly rectilinear, and may impinge upon the solid confines of the bodies. Thus would ensue a violent consolidation of the pair.

Thus, in the illustration of the cannon-ball, velocities at

periastron varying only between about 26,000 and 37,000 feet per second would permit permanency of orbit. Anything higher would lead to dissociation; anything lower to collision. If, when velocities were between these critical conditions, the radial component only of motion should be abstracted, the orbit would reduce toward a circle of large radius about the earth, at a rate of 26,000 feet or less per second. The speed mentioned would then be the one of equilibrium, and the pair would exist permanently, as a "unit," in this condition, until again disturbed from without.

But this process of abstracting energy radially could never be carried to the point where the orbit became truly circular, with zero eccentricity. For energy might be abstracted radially only to the extent that a radial component of motion were present. As the orbit approached circularity radial energy could be abstracted only with greater and greater difficulty, or under more and more unusual conditions. The process could not imaginably proceed until all eccentricity were gone.

If, on the other hand, the energy were abstracted tangentially at apastron, or at any rate during the outer half of the elliptic orbit, the latter would become a more narrow ellipse, like that of any cannon-ball, and collision with the earth would ensue quite as in any such a case.

These actions can perhaps be better understood from Fig. 5, which displays the several possible orbits of a body M_1 relatively to a larger mate. Around the mate is shown a dotted circle ZZ, the radius of which is the sum of the radii of the (supposedly spherical) bodies. Any orbit which touches this circle will of course end in collision. Thus, AA and FF are both hyperbolic orbits; but one of them will end in dissociation, the other in collision.

Should *radial* energy be abstracted from AA by radial means, between P and A, the orbit would be altered to some hyperbola or ellipse of less eccentricity than AA and passing through the point M_1; that is, the mean energetic distance would be conserved. The limiting case for such a process, when all the radial energy had been withdrawn, would be the circle, which is the only one of these orbits of lesser eccentricity which is shown.

The withdrawal of *tangential* energy from AA would lead to

a hyperbola of increased eccentricity, passing through M_1 and cutting the circle ZZ.

At periastron P the radial energy of AA (as was pointed out in an earlier paper) is all kinetic in form; and that kinetic energy is directed tangentially. Yet all of it above the velocity for circular equilibrium at this radius is true radial energy; for

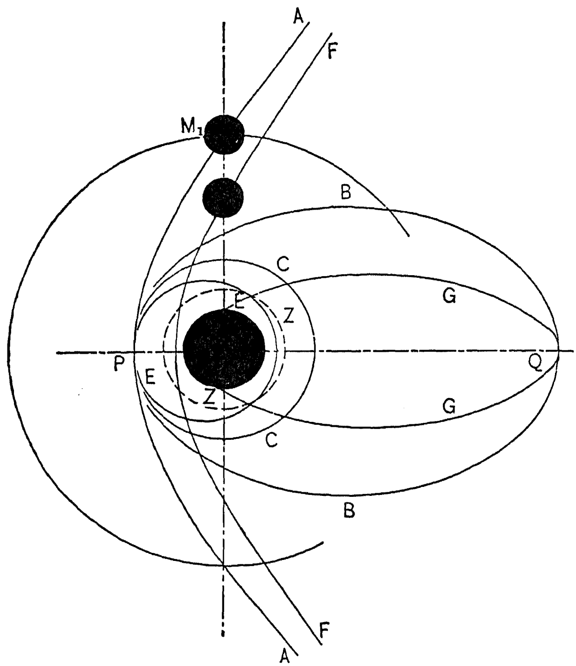

FIG. 5.

its radial or centrifugal force is unbalanced, and the motion-energy will quickly become radial in direction as well as name. Therefore, if kinetic energy be abstracted at P, it amounts to a reduction of radial energy. Radial energy will have been reduced, though not by radial action. The eccentricity will decrease and the orbit finally become an ellipse, such as BB. Further abstractions of kinetic energy at P will alter the orbit to a circle, such as CC, or an ellipse of reversed eccentricity, such as EE, which ends in collision.

The abstraction of kinetic energy at the apastron Q of BB,

on the other hand, increases the eccentricity, but decreases the periastron distance, as in the orbit GG, and leads to collision. For at Q the kinetic energy is all tangential energy, the radial energy being space-energy.

Now in the illustration of the cannon-ball the energy was all supposed to be supplied at periastron, as at P for the orbits CC, BB or AA. But in the use for which these arguments are designed, as mechanical similes for molecular action, all contributions or abstractions of energy will be by other systems which are external to the one in question. The point of contact between the two systems will be at or near apastron, such as Q for BB or GG, or P for EE. The only way in which tangential energy can be exchanged is tangentially and kinetically; and even then it can be exchanged only through the medium of conversion into or from radial energy.

Radial energy, on the other hand, cannot only be exchanged directly with external systems, but it can be exchanged in two distinct and different ways. These ways are (1) *spacially*, by radial action *between* periastron and apastron, and (2) *kinetically*, by tangential action *at* periastron or apastron. Referring to the illustration of the cannon-ball, the first would be instanced by the interaction of the projectile, during its outward or inward flight, with some mass which was itself moving outwardly or inwardly, relatively to the earth. The second would be instanced by the original impulse of the projectile, by the gunpowder, or by some similar, but negative, influence experienced from some third mass at the extremity of its flight away from the earth.

This question of the possible ways by which energy may be gained or lost by a mass-pair should be clearly understood. They may be stated briefly as follows, although their significance will not come out until the discussion reaches the question of the mechanical theory of heat. Thus, energy may be gained or rejected by a mass-pair in three different ways, each of which will have a different effect upon the form of orbit and upon the chances of its alteration past one of the critical points of intensity.

1. Energy received tangentially at apastron (as in the illustrative cannon-ball before its trajectory had cleared the earth) increases the mean energetic distance D and decreases the eccen-

tricity e. Energy imparted tangentially at apastron, of course, reverses this rule.

2. Energy received tangentially at periastron (as in the illustrative cannon-ball after its trajectory had cleared the earth) increases both the mean energetic distance and the eccentricity. Energy lost in similar fashion reverses this rule.

3. Energy received radially, between one extreme energetic condition and the other, increases the eccentricity, but does not affect the mean energetic distance. Energy expended radially decreases the eccentricity without affecting the mean energetic distance.

In the illustration of the cannon-ball, by which the natural action of a free projectile was sought to be made clear, there was used, it is true, an original impulse, viz: the energy of the gunpowder, which is foreign to the simple interaction of two mass-portions by gravitation and inertia. Yet the illustration may not, for this reason, be outlawed; for in the interaction of a projectile with another mass-system consisting of more than one mass-portion there may arise a form of impulse which closely resembles that of gunpowder and cannon.

If we revert to the simple energetic system displayed in Fig. 2 and imagine M_2 to be, instead of a doughnut-like solid, a ring of separate solid mass-portions, revolving about the center C in a swarm, in circular orbits, then the situation becomes more complex, and capable of activities beyond those already described. Let it further be imagined that M_1, instead of following the straight line ACB back and forth, follows some conic-section orbit which carries it periodically through the point C.

Now the intensity of action at this point was found to be proportional to $\frac{1}{S_0}$, wherein S_0 is the minimum distance of approach between M_1 and M_2. But when M_2 is an annular body such as shown in Fig. 2, the minimum value for S_0 is the radius of the annulus. Therefore, if the annulus should contract during any of M_1's circuits of its orbit, the intensity of action at its next approach to M_2 would be accentuated. The energy released from M_2 by the mutual approach of its members would be transferred to M_1, imparting to it a greater velocity; and this increase, occurring at periastron, must be radial in its character.

It will go to increase both the mean energetic distance and the eccentricity of orbit between M_1 and M_2.

If the argument be transferred to Fig. 3 new possibilities appear. If M_2 thereof should consist of two or more portions, held apart by their relative motion in elliptic orbit, the degree of propinquity at periastron, or the intensity of energy of the M_1M_2-system, would depend upon whether the two or more orbits all reached periastron simultaneously or not. Should M_2 be in its condition of maximum separation of parts when M_1 approached, the intensity of action would be slight. But if all orbits coincided in phase, reaching greatest propinquity simultaneously, the intensity would be very much greater. The impulse received by M_1 in any such a way would be quite the parallel of the impulse received by the illustrative cannon-ball from the gunpowder, at periastron.

Thus, from period to period of the M_1M_2-system the intensity of its radial energy might vary widely. If M_1 were the messenger carrying a manifest of the system's energy to foreign parts, which could not directly perceive the more massive, tangential and torpid energy of M_2, the system would be observed thereby as embodying widely varying intensities of energy. While at one time the intensity might be so limited as to confine M_1 to an elliptic orbit, at another it might project it with an hyperbolic orbit. Or, conversely, the projectile M_1, having entered the system from foreign parts on an hyperbolic orbit, might be entrapped and remain in elliptic motion. Indeed, its own arrival might be the cause of such dissociation between the parts of M_2, at the expense of M_1's energy, that the latter no longer possessed sufficient energy to get away again.

Of the portions of the orbit further from periastron, those beyond the mean energetic condition are most illustrative; though all that is said also applies to points inside the mean.

In its outer portions the orbit may be perturbed by the approach of some third mass-portion arriving as a messenger from more distant fields. This third projectile will possess a motion aimed more or less directly at the mass-center of the M_1M_2-system. To the extent of its component thus directed it will exchange *radial* energy with the system. To the extent of its component normal to this direction it will exchange *tan-*

gential energy. The effects of these exchanges upon the form of the original orbit have already been noted.

It is not possible to trace here all the varied possibilities of such interchanges of energy, even when occurring between systems of only three or four mass-portions. What is desired is merely to impart an elementary concept of the distinctions between radial and tangential energies, and the ways in which these may be affected by other systems without infraction of the Conservation of Energy. It is now plain that such outside influences might alter widely the intensity and eccentricity of any energy-system, leading it toward or over either of the two critical limits of intensity as defined above. It will be undertaken later to show that such outside influences may also affect just as widely the extensity of a mass-system, though the process is not quite so simple and obvious.

This completes the bare statement of what mechanical energy is, and by what methods only it may be imagined as augmented or diminished. Whatever hypotheses as to other forms of energy than mechanical may be made, if the latter are to be regarded at all as mechanical energy in disguise their activities must be brought into line with the preceding analysis.

For the present it suffices to point out that there has already resulted from this analysis a useful classification of mechanical energy-forms, which may be given a habitation and a set of names in the following table, as "permanent," "subpermanent" and "superpermanent" types of energy, respectively.

1. The *permanent* forms of mechanical energy are those embodied in elliptic orbits of sufficient dimensions to clear the solid confines of the two bodies. The mathematical conditions defining this class are that the eccentricity e shall be less than unity and the periastron distance S_0 shall be greater than z, when z is the sum of the radii of the two solid masses.

2. The *subpermanent* forms of mechanical energy are those embodied in orbits, either elliptic or hyperbolic, which pass within the solid confines of the bodies and end in collision. The mathematical conditions defining this class are that e may have any value whatever, but S_0 must be less than z.

3. The *superpermanent* forms of mechanical energy are those embodied in hyperbolic orbits only, when of sufficient

periastron distance to clear the solid confines of the bodies. They end in dissociation, instead of collision. The mathematical conditions defining this class are that e shall be greater than unity and S_0 greater than z.

It will develop later that, whereas all the so-called "mechanical" energies of the engineer must belong solely to the second of these three classes of energy, the molecular or atomic or electronic energies, which we call heat, chemical energy, electricity, etc.—if they can be regarded as modes of mechanical motion at all—must belong to the first and third types. As to electrical energy there is more doubt, because lack of permanence is its chief characteristic; yet in electrical matters all questions of time must be referred to such exceedingly minute units that it may appear, upon examination, that electricity, like light, is in reality one of the most permanent of all forms of energy.

In the distinctions portrayed in Fig. 5, therefore, we are on the edge of understanding that most wonderful and significant of all natural phenomena, the *transformation* of energy. For, aside from the contrasts between the kinetic and potential, and the radial and tangential—sorts which form a part of every type of energy—no alterations in form of action have developed, thus far in the analysis, which are so subversive of external appearance as are these changes from permanent to sub- or superpermanent conditions.

The dividing lines between the permanent, as the central form of energy, and the sub- and superpermanent forms on its either hand, are called the *critical energetic conditions*—the lower and upper critical intensities of energy, respectively. These critical limits have already been briefly defined in this chapter. The part they play in energetic phenomena cannot be discussed until other forms of energy than the mechanical are discusssed.

The upper critical intensity has long been recognized and taught—although chiefly of interest in astrophysics—as the "critical velocity." For each mass-portion, it has been taught, there exists a certain velocity for smaller mass-portions, above which the latter would dissociate from the former. If this idea be made more accurate by defining the critical function as a certain value of $\frac{V^2}{M_1+M_2}$, instead of merely velocity, and

broadened by recognizing it as an intensity of motion-energy, it can remain for present purposes unchanged.

The lower critical intensity, on the other hand, has not been similarly recognized and taught, so far as the ordinary text-books give evidence. It is directly proportional to intensity of space-energy, or degree of propinquity, or solidarity, $\frac{1}{S_0}$, according to the several ways in which it might be named. For every mass-pair of specified density of members, therefore, there is a certain degree of intensity of concentration (which is itself a form of energy) which cannot be exceeded without entailing collision and energy-transformation.

In all of this discussion it must have become long since obvious that, in the natural, mechanical interaction of mass-portions, the straight line plays a very subordinate part, if it appears at all. As a matter of fact, it does not appear at all. It has already been clearly shown how, mathematically speaking, the straight line constitutes one unattainable limit of eccentricity of orbit, on one side, while the circle constitutes another equally unattainable limit of eccentricity (namely, zero eccentricity) on the other. Centrally between the two lies the PARABOLA, with unit-eccentricity, constituting the geometrical fundament of natural energetics. It would be the next step of development of this question of the absorption and rejection of energy by mass-systems to show that this mathematical aspect of the situation is also the natural one—except that in mathematics limits are attainable, whereas in nature they are not.

For it is a fact, in the natural aspect of the question, that the more radial energy a system possesses, the more readily it will reject or impart energy to other systems, and the less readily it will receive it in radial form. Conversely, the more tangential energy a system embodies the more readily it will receive and absorb radial energy, and the less readily it will impart it. It follows, therefore, that the smaller the eccentricity becomes, the greater difficulty is there in reducing it further, and the greater are the chances that the eccentricity will increase, in any energetic mix-up, rather than still further decrease. Conversely, the greater the eccentricity becomes, the greater is the likelihood of

its being decreased, in any energetic encounter, rather than being still further increased.

The result of this view of the case is to place the medium value for the eccentricity—unity—in the most conspicuous position, as the value forming a center of stable equilibrium, on either side of which the variations in eccentricity swing, as does a pendulum about its vertical position. Just as it was found (see Chapters II and III) that both velocity and spacial separation, in energetic systems, swing in stable equilibrium on either side of mean energetic values for both variables—on either side of a mean energetic distance of separation and a mean energetic velocity, neither of which could ever attain to either zero or infinity—so now it appears that the eccentricity of natural orbit also varies on either side of its mean energetic value.

This mean energetic eccentricity of orbit is unity, defining the parabola. This is the mean or average energetic condition of every mass-pair in the universe. A mass-pair in this mean condition of eccentricity would be just upon the line between confining its energies at home, in stable permanence, and sending them abroad. From this mean the eccentricity may be reduced into elliptic motion by the *abstraction* of energy from the system. But such abstraction of energy becomes more and more difficult as it proceeds, and no imaginable natural conditions may ever be defined which would succeed in reducing the eccentricity to zero, in circular motion.

Similarly, the *absorption* of energy by any mass-pair having the mean energetic eccentricity of orbit, or following a parabolic orbit, may increase that eccentricity into hyperbolic motion indefinitely. But the difficulty of further absorption of energy increases as it proceeds, and no imaginable natural conditions may be defined which would succeed in expanding the eccentricity to infinity, or developing straight-line motion.

It is therefore obvious that the teaching of mechanics to mature students by basing everything upon straight-line motion is unnatural to the last degree. The fundament of all true mechanics is parabolic motion. That stands to all other forms of orbit as our sun stands to all possible vagaries of its innumerable family of satellites—as a natural base and center of equilibrium which, albeit itself unsupported and undefined in space, may yet never be disregarded as the natural starting point for all discussion.

CHAPTER VI.

THE GENERAL NATURE OF MECHANICAL ENERGY.

The definition of mechanical energy is now complete, so far as a definite, though skeleton-like, structure is concerned. But this skeleton needs clothing with some flesh and form, before it may be useful for a display of the nature of heat.

The outline of the skeleton, to summarize for convenience, may be stated as follows:

1. Energy has been identified as always consisting of the arithmetical product of two variables. One of these variables has been named the *intensity,* and the other the *extensity,* of energy.

2. Intensity has been shown to be a function of either the *space*-relationship or the *motion*-relationship between two or more mass-portions. The intensity of space-relationship is proportional to the "propinquity," or the reciprocal of the distance of separation. The intensity of motion-relationship is proportional to velocity-squared-divided-by-aggregate-mass-involved.

3. Extensity has been shown to be the measure of the amount of *mass-pairing* involved. It is proportional, other things being equal, to the square of the total mass involved. For any given total mass it increases, but not proportionally, with the degree of subdivision of that mass, into mass-pairs capable of embodying the relationships defined above.

4. It was shown that energy-quantities may vary by variations in either intensity or extent. In the applied mechanics of engineering it is only the intensity-factor which varies appreciably; that is to say, we vary space or motion, while the mass-factor remains proportional to the mass involved. But in thermal energy or other intricate forms, when viewed mechanically, the extensity-factor must be expected to vary as often and widely as the intensity; that is to say, the energy, its intensity being fixed, is no longer necessarily proportional to the mass involved.

5. The variation of any mass-system in intensity of energy may take place smoothly, in stable equilibrium, within a certain

range. This range is defined at either end by the two critical intensities. Trespass over either critical intensity causes the equilibrium to become unstable. Trespass over the lower, or spacial, critical limit of propinquity, or $\frac{c}{z}$, leads to collision. Trespass over the upper, or kinetic, critical limit of intensity, or $\frac{V^2}{M_1 + M_2}$, leads to dissociation. Either collision or dissociation constitutes a *transformation* of energy.

6. Mechanical energy existing in stable equilibrium, *between* the critical limits of intensity, may be called "permanent" in form. That embodying intensity greater than the *spacial* limit of propinquity has been called "subpermanent" in type. That embodying *motion* above the upper critical limit of kinetic intensity has been called "superpermanent." Of the latter two, the first type can exist only throughout a portion of a single revolution, and is perceptible to the human senses only when the members of the mass-pair are large enough, and the period of revolution long enough, for their separate observation. Such is the case in celestial mechanical energies, and in the applied mechanics of machines.

7. It is next to be pointed out—and this is one of the most important steps in the understanding of energy—that the energetic conditions of matter, whether spacial or kinetic, whether referring to intensity or extensity, never spring from, nor are measurable from, an *absolute zero* of any one of the factors involved. Instead, the factors in any energetic condition vary on either side of a central, or *mean energetic,* value; and this value itself hangs self-supported in space, so to speak, with no means known for referring it to any absolute base. It is to an explanation of these statements that Figs. 6 and 7, and the next few paragraphs of discussion, are to be devoted.

The earlier papers of this series defined the intensity of energy as proportional to $\frac{1}{S_0} - \frac{1}{S}$, when potential, and of $\frac{V^2 - V_0^2}{M_1 + M_2}$ when kinetic. The law of the conservation of energy links these two forms, so that either may be studied as a representative of both.

Energy may consist either of little space and much motion

(or force), or of much space and little motion (or force). B
these paired quantities appear, not as a *sum,* but as a *produc*
If they appeared as a sum, either of them could be reduced
zero at times (the quantity of energy remaining constant, accor
ing to the conservation of energy) by a sufficient growth
the other. But being bound together as a product, *neither fact
may be reduced to zero by any finite growth, however great,
the other.* And since "infinities" apply only to portions of t
universe so large as to exceed human understanding and mea
urement, and therefore have no place in any exact natural sc
ence, zeros are likewise excluded from participation in energet
phenomena.

Thus, as space disappears, in nature, energy of motio
appears in proportion to $\frac{1}{\text{space}}$, and force appears in proportion
$\left(\frac{1}{\text{space}}\right)^2$. Therefore no finite accumulation of energy or forc
however great, can ever make the space zero, or compress matt
into nothingness. This agrees with our most ordinary concep
of matter; for two of the prime attributes of matter, defined
elementary in the earliest study of nature as an exact scienc
were its indestructibility and its occupancy of space.

On the other hand, as space appears, force disappears, an
energy is absorbed. Yet no imaginable degree of finite spac
can ever reduce the force to zero, or quite annul the absorptio
of energy with further increase of space. This concept dat
from Newton's discovery of the law of gravitation, and lies
the heart of our modern concept of the universe as a unit—i
every part bound inseparably to its every other by an unbreal
able, albeit a very elastic, bond. This concept is most famili
to engineers in connection with gases and vapors, which expan
indefinitely, losing pressure as they go, yet with no possibility
the pressure ever reaching zero.

As for velocities, they must follow the same general law
although in accordance with a different mathematical functio
Whereas force is proportional to the square, and energy to th
first power, of the propinquity, or the reciprocal of spac
velocity is proportional to the square root of that same functio
Although the rates of variation would therefore differ in thes
three cases, yet the general form of the relationship remains th

same. No velocity can be so great as to reduce the space to zero, and no space so great as to reduce the balancing velocity to zero.

This general form of relationship between force, energy or motion, on the one hand, and space on the other, is shown in Fig. 6. The curve would not be the same in the three cases, but it would have the same general form; and because of the difficulty of making one scale show all three functions to advantage, one only is shown to represent them all. The function is seen to be a curve asymptotic to the two rectangular axes. Each factor may vary as widely as it pleases, and may thereby vary the other. But neither can ever force the other to zero, by increasing ever so widely.

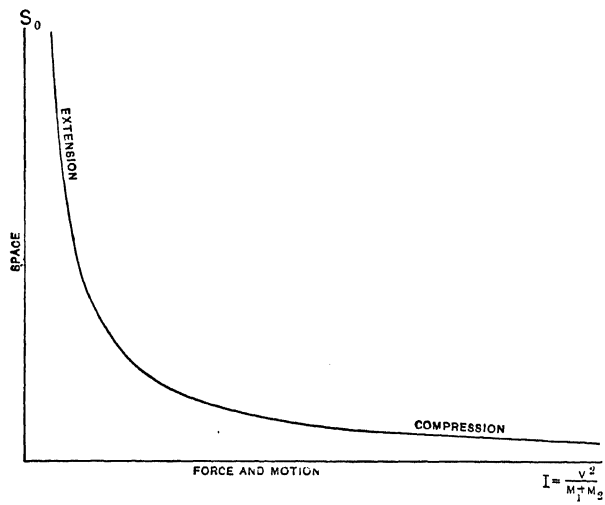

FIG. 6.

Moreover, as either may seek to force the other to increase, by itself decreasing toward zero, it will find itself working against an increasing mechanical disadvantage as it proceeds. The further it goes the greater is the proportion of resultant to creative action. By the principle of virtual velocities, further progress must become more and more difficult with each advance. The tendency is always to return toward a central or medium value for each of the factors. When space becomes deficient and the force excessive, force tends to control the situation; as a compressed spring or gas tends to burst its bonds. When

space becomes excessive and force deficient, space tends to rule the game; as when an elevated weight tends to fall, or a distended gas to be condensed by external pressure. The natural equilibrium in which these factors vibrate on either side of their central, or mean energetic, values, in either direction, is thoroughly stable.

This same general form of energetic relationship and stability of equilibrium applies also to the other energetic variables, as well as to space, force and motion. Thus, in Fig. 7 can be seen the way in which the extensity-factor, or quantity-factor, of energy varies in terms of the degree of subdivision of the aggregate mass embodying energy of any stated intensity. In the Fourth Paper was developed the equation for this relationship, in Equation 26, which is repeated here

$$X = \tfrac{1}{2}M^2 \left(1 - \frac{1}{n}\right) \qquad (26)$$

for convenience. In it X is the extent of mass-pairing, or extensity of energy, which is embodied in the mass M by its subdivision into any number n of equal parts.

The variation of X with n, for any given mass, is seen in Fig. 7. When $n = 1$, or the mass is a homogeneous, solid unit, embodying one arbitrary unit of mass, X=0 and the function appears at A, Fig. 7. When n=2, $X = \frac{1}{4}M^2$ and the curve passes to D, upon a scale determined by the size of the mass-system or the arbitrary mass-unit in question. But as n increases still further, X exhibits an increasing slowness in following proportionality to it. A doubling of n to 4 increases X by only one-half. A quadrupling of n to 8 increases X by only three-quarters; and no finite extension of the value of n, however great, can quite succeed in doubling the value of X from the point D.

When the arbitrary unit of mass which forms the measure of each "equal" portion becomes greater than one-half the total mass—that is, when the aggregate mass is divided into only two *un*equal portions, the larger one of them constituting the unit of mass and the fractional remainder—n may have values (always positive) which are less than two. Such would be the case in engineering mechanics, where from the total mass of the earth only a small fraction is split off, made into a hammer or a cannon-ball or a locomotive, and its energy relatively to the

remainder (which we still call "the earth") utilized for human purposes. In such case X would become a small fraction of M^2, and the curve of Fig. 7 would pass from D toward C. When the total mass present amounts to just one unit of mass, X becomes equal to zero. When the total mass present is less than one unit of mass, n becomes less than unity and X becomes negative.

As the left-hand limb of the curve approaches the condition of a straight line parallel with the axis, the degree of mass-pairing, or the extensity of energy, approaches proportionality with the mass of the smaller fragment. Absolute proportionality is what is assumed in the equations employed in engineering.

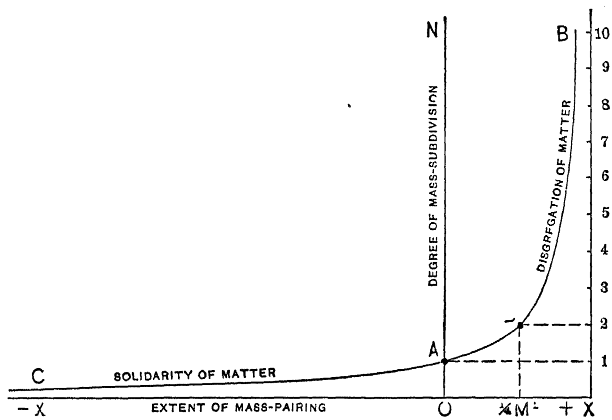

FIG. 7.

But it is plain from Fig. 7 that this assumption could become true only in the impossible case when n became zero, when the fragment split off from the earth became zero and the extensity of energy became minus infinity—for only then would the limb AC of the curve BAC have become a straight line.

It therefore becomes plain that the extensity of a mass-system, or its capacity for embodying intensity of energy, varies, with the fineness of its subdivision into separate portions, quite as does space with motion. The relationship swings on either side of a central, or mean energetic, condition, or arbitrary zero, such as D, Fig. 7. No absolute zero is attainable in either direction. Even if the axes which may be said to measure absolute zeros, at the foot and right-hand, respectively, be

regarded as basis of convenience which it would be well to retain, the fact constantly to be kept in mind is that the energetic condition never passes to either of them, and never can. This general characteristic holds true, whether expressed in terms of intensity or extensity of energy, whether of space, motion, force, degree of massive solidarity on the one hand, or of fineness of comminution of mass on the other. Any of these factors may pass to either very great or very small values, but none of them may ever attain to either zero or infinity.

Indeed, it will appear, as the argument proceeds, that every energetic relationship which can be stated exactly follows this same general law. To those engaged in power-engineering the most familiar illustration of this statement is the hyperbolic relation between the pressure and volume of any gas. Pressure and volume always appear as a product, each being inversely proportional to the other, or to some power of the other. The general equation is $PV^x = $a constant. They never appear as a sum, one decreasing as the other increases. No degree of pressure, however great, can ever reduce the volume of any gas to zero; nor can any degree of expansion, however great, ever reduce the pressure to zero. There is no place in the universe where the pressure or density or volume of elastic matter is imaginably zero.

In every case, all energetic functions are founded upon a central, or mean energetic, condition, which hangs unsupported in space, so to speak, as the sun hangs in the heavens. No absolute base or support for it is imaginable or necessary. It is on either side of this central point of reference, and not up and down from any absolute zero, that all energetic factors vary. These statements, which have been made in reference to mechanical energy only, will be found to apply universally.

Energetic Equilibrium. In all natural phenomena the one most important guiding principle, after conservation, is that of universal stability of equilibrium. The determination of what shall be the next in that most intricate series of occurrences to which we give the general name, the progress of events, always depends upon stability of equilibrium. The natural universe is always, except locally and temporarily, in stable equilibrium. And if its equilibrium temporarily and locally has become unstable, the movement is always toward the recovery of stability.

Whatever may occur in the nature of a departure from the general medial trend of affairs always brings with it, as its immediate consequence, a tendency to departure in the countervailing or balancing direction. Although this tendency may not prevail immediately, it must ultimately.

This law has its foundation in these elementary mechanical systems now under discussion. They were likened, in the opening pages of the second paper, to the familiar pendulum, which is seen to swing always in stable equilibrium. Turning from that to the less familiar, but only true, energetic element, the two-part free mass-pair, the same truth appears. The element swings in stable equilibrium between two extremes, one of unusual space and the other of unusual force or motion. In this swing, departure in either direction begets increasingly a tendency to return. The attainment of unusual space kills the motion which begot it, and increasingly invites motion of return. Unusual lack of space begets both force and velocity, and tends increasingly to a reversal of motion and a recreation of space.

The same law applies to the mass-pairing factor of energy. It varies on either side of a central value, of a mean average size of solid or undivided mass-portion, in stable equilibrium. The unusual consolidation of any number of the mass-portions of a system begets unusual disgregative velocity in the remainder. This of itself constitutes a dispersion of matter. But in addition, the unusual velocity of this remainder, returning in due time, tends to beget a renewed separation of the originally consolidated group. As much as this can be seen in pure mechanics, with collision and heat-formation excluded from the discussion; but so soon as these phenomena are admitted, as will be done in the next chapter, the field of this form of stability of equilibrium will reveal its extension into other forms of energy, in a most beautiful way.

The same law applies to the eccentricity of orbit. Unusual eccentricity tends to impart energy radially from the system to outside bodies; and the loss of this energy tends to a reduction of the eccentricity. Unusual lack of eccentricity, on the other hand, invites the absorption of energy contributed radially from other systems; and the effect of such absorption is necessarily to increase the eccentricity.

Obviously, too, this phenomenon cannot proceed to any rigid

or abrupt limits. Eccentricity of orbit can never reduce itself quite to zero, by the radiation of energy; because the ability to do so depends upon the presence of the eccentricity itself. Circularity of orbit cannot absorb radial energy to the point where the eccentricity is infinite, because the ability to absorb is lost as the eccentricity increases.

The fundamental law of this equilibrium, as evinced between eccentricity, mass and dimension of orbit, is based upon the conditions found to prevail in our solar system; wherein the few score bodies, the motions of which we can study, have had time, since the dawn of astronomy at least, to settle into stable equilibrium. The existing state of affairs is defined in the equation derived independently by La Place and Lagrange, viz:

$$\frac{M_1 M_2}{M_1+M_2} . e^2 \sqrt{S+S_0} = \text{a constant*} \qquad (28)$$

This is not an equation of energy-interchange, but one of fact, showing the effect of centuries of energy-interchange. It is incidental to and illustrative of our argument, rather than basic for it. But it is of more than incidental significance that this equation reveals the same general relationship between the factors of energy as those given previously. The three factors of mass, space and eccentricity are linked together, not as a constant sum or a constant ratio, but as a constant product. Any one of the three remaining constant for the time, either of the other two can vary to an unlimited degree; *but only as the reciprocal of the third.* Neither can be brought to zero by any expansion, however great, of the other. Neither can approach zero without encountering increasing resistance, in the unusual expansion of the other which must accompany it.

In all of these respects, the elementary free mass-pair does not constitute for engineering students a forcible illustration, for here on the earth's surface we have no free mass-pairs big enough to be seen. All that we have which are free are of molecular dimensions, and our knowledge concerning them is chiefly inference. But as consideration turns to the forms of energy other than mechanical, it will appear that all energetic

*The writer is uncertain whether the first factor in this equation should be as printed, or simply M. Consistency with all the other true equations of mechanics would give it the form here printed. But, like all these other basic equations, it is to be found under the cognisance of high authorities in terms of simple M.

action follows, in a most striking way, these same general characteristics. All of them swing constantly on either side of a mean energetic condition, against resistances which increase as the departure from the mean condition increases, between limits of zero and infinity neither of which can ever imaginably be reached. It is of vital importance to the mechanical theories of heat and these other energy-forms, therefore, that these same basic characteristics be noted as attributes of the most elementary, mechanically energetic mass-pair.

Energy-transformation. So far as the mathematical forms of the curves connecting the several factors of energy are concerned, these swings of energetic condition on either side of the central mean might extend indefinitely, along the asymptotes to either axis. But when mathematics is replaced by observation of natural fact, it appears that each curve fails of continuity, if pushed too far along its asymptōte. Some factor hitherto irrelevant enters and controls the situation. The energetic equilibrium, stable up to this point, becomes abruptly unstable. Smooth interaction at a distance between the two mass-portions comes to an end. Either dissociation enters, to put an end to the identity of the mass-pair as a perceptible pair, or collision enters to put an end to the conservation of the original form of energy.

This, then, is *energy-transformation,* the break in the continuity of the curves of stable equilibrium and of visible conservation of energy which reveal the critical limits of intensity of energy—the critical limits to the concentration of energy in, or of abstraction of energy from, a mass-system of the particular degrees of mass and of mass-pairing in question.

What ensues then is more difficult to explain than what has preceded. We know now, from considerations broader than any yet permitted to enter the argument, that trespass beyond these critical limits of intensity abrogates neither the Conservation of Energy nor the universal Stability of Equilibrium. It is only in terms of the particular form of energy in question—in this case mechanical energy—that the continuity of conservation and stability is broken. The line is then crossed which arbitrarily defines this energy-form from the others; and across this line, with the energy, we must step, if we are to follow clearly the continuity of universal natural action. When we have crossed we shall see that what we have crossed was indeed an arbitrary line,

like a state boundary-line, erected in the human imagination to serve the convenience of human limitations; but having no other real existence. We shall see that, so far as light now penetrates, all energies are one in their fundamental components.

The sole trouble is that the light does not penetrate far. In other energy-forms than mechanical we cannot see these component parts. We know these more obscure energies only by their blanket results. Yet if we are ever to get any more clear and concrete idea of their anatomy, it seems inevitable that it should be in terms of mechanical energy. Certainly the engineer and the engineering-student, if not all others, can proceed more clearly from concepts based upon mass, space, force and motion than they can by relying solely upon abstract empiricisms, stated mathematically. The writer will attempt no argument that heat is or is not a "mode of motion." He believes that it is. The evidence that it is, albeit vague and inconclusive to some minds, is too great in volume for denial. We should lose too great a portion of our scientific perceptions if we should deny the mechanical nature of heat, and do it consistently. But that is not the point. The point is that *if* heat be truly a "mode of motion" and chemical energy truly a "mode of arrangement" of mass in energetic action, our concepts thereof must be guided by the fundamental principles of mechanical energy which have been displayed above. We are now, for the first time in the argument, mechanically equipped for an accurate pursuit of the questions: *What* mode of motion? *What* mode of arrangement? among the many imaginable ones, may or must they be?

CHAPTER VII.

What is Heat?

In asking ourselves what is heat the most surprising thing is to think that not every one, that possibly no one, knows what heat is. Heat is one of the commonest things in the world. It is as common as matter; for we know of no matter without heat. It is as common as space; for while space cannot embody heat, yet no space is known which does not contain either matter or else that radiant energy (commonly called "light") which, traveling always at the inconceivable rate of 186,000 miles per second, turns into heat as soon as it meets solid matter.

Yet we have no definition of heat. Heat seems to be many different things, according to how it is encountered. It has been called "the waste-heap of the universe." Indeed, it seems to be easier to say what heat is not, than what it is. Heat is not matter. That point was settled a century ago. It is a "form of energy." But all that that means is that it is capable of performing work. But as it is capable of many other things besides performing work, and as many other things besides heat are capable of performing work, this is not a very satisfactory definition.

Heat is like an ant-colony. It lives in a hidden nest. We can see it go into and come out of its nest—matter—by several doors. But as to just whither those doors may lead, and what may be the form of structure which connects the several doors, is as yet pure surmise.

Fortunately, there are quite a number of doors to the thermal ant-hill; and as heat appears at each of these it bears a different guise. So that, aided rather than hindered by the very diversity of the problem, we are able to guess fairly near to the sort of interior arrangement which alone could fit all of the doors.

Heat appears in and disappears from matter by the following processes:

	Methods of Heat-gain.	Methods of Heat-loss.
(1)	Conduction from hotter bodies.	Conduction to colder bodies.
(2)	Absorption of radiation.	Radiation to colder bodies.
(3)	Impact and friction.	———————
(4)	Compression.	Expansion.
(5)	Combustion.	Dissociation.
(6)	Electrical resistance.	Electrical generation (by thermo-pile).

There are other thermal processes than these, but they occur upon too small a scale to be of present interest.

Of all of the above processes the two most familiar sources of heat are radiation (sun-heat) and combustion. But both of these processes are complex and obscure, when viewed from the stand-point of the present articles, which are to concern themselves with a mechanical explanation of heat. Sun-heat is a transformation of radiant energy, and combustion a transformation of chemical energy, into heat; and both radiant energy and chemical energy are just as much in need of an explanation as is heat itself.

Mechanical work is the only form of energy of which we now have any definite and clear concept. It is by the doors opening between that form and heat that the latter must be approached. These doors are the processes numbered three and four; and of these Number Three comes first, both numerically and naturally.

But the question of impact and friction can be broached for discussion only in terms of elasticity and its opposite.

Elasticity and Inelasticity. When two solid bodies come into contact the collision is always partially elastic and partially inelastic. That is to say, a part of the kinetic energy inherent in the bodies before collision is returned, in the form of motion in the reverse direction, and a part is not. In so far as the energy is returned kinetically, the bodies are said to be elastic. In so far as it is not, they are said to be inelastic. While some bodies are almost perfectly elastic, and others almost wholly inelastic, none are known which are completely either.

In so far as bodies are elastic, their collision can have no effect upon the mechanical principles laid down in the preceding papers. Two bodies engaged in a mutual orbit which brought them into collision would, if perfectly elastic, rebound with a velocity as great as that before collision. Only the direction of motion would be altered. The original conic-section orbit would

be continued unaltered, except that its new axis would be inclined with its old one. The mean energetic and critical conditions would remain at the same intensities; but the collision which constituted the lower critical intensity would not lead to a transformation of energy, as when inelasticity is present.

Elasticity, however, while incapable of throwing any light upon the principles of motion, throws considerable light upon the energetic nature of mass—at least, in the form of the so-called "solid" bodies. For elastic collision means the temporary storage of the energy of collision in the deformation of the bodies, against their disposition to retain their solid form; which stored energy is given out again in the rebound, as the original forms are regained.

But, if our ideas are to remain true to the elements of mechanical action as stated by Kepler, Newton and La Place, as collocated in the preceding papers, this temporary storage of energy within each body cannot be attributed to pure mass. It is only in changed relationships *between* mass-portions that energy can be stored. Each of the elastic colliding solids must therefore be regarded, not as a unified or truly solid portion of mass, but as a more or less complex system of mass-portions, between which the energy may be stored. The way in which this may be done is not the point of immediate interest. The significant fact is that no body which exhibits any elasticity whatever, may be regarded as a solid unit. *No truly single or homogeneous body, whether it be of the magnitude of a moon or a molecule or an electron, can possess elasticity, any more than it can possess energy.*

Elasticity can be an attribute only of the *subdivision* of mass. Wherever the mind may be disposed to chase that most elusive concept, "the ultimately indivisible portion" of truly solid or homogeneous mass, the one quality which must be assigned to it is that of perfect inelasticity. The attempted concept of an ultimately indivisible, yet perfectly elastic, "atom" is as inconsistent with all accurate scientific experience and principle as is the concept of perpetual motion. Both concepts have arisen from the desire for a royal road of unnatural ease to the solution of natural problems.

As for inelasticity, that brings the discussion home to the question of what is heat; for inelasticity is merely a short name

for the degree to which kinetic energy is converted into heat by impact, when bodies collide.

In order to get the problem into simple form, let it be supposed that the colliding bodies are perfectly inelastic; for the addition afterward of that modicum of elasticity which is always present in fact will not affect our conclusions as to the portion which is inelastic. Now the word "heat" being merely a subterfuge, or cloak for ignorance, with which we cover up the fact that the energy disappears and we do not know what form it takes, let heat be excluded from the discussion. We may refuse to use the word until we have an exact idea to attach to it.

What form, then, may the energy of the colliding bodies take, if both elastic rebound and the formation of heat are excluded, and the conservation of energy is still to hold true? Only one, by any possibility, viz: *the rupture of the bodies and the separation and scattering of their fragments.* This is the only truly mechanical process, aside from elastic rebound, which will absorb energy inelastically.

But, if the bodies collide as free bodies, uninfluenced by the propinquity of other and greater masses, such as the earth, the fragments will not stay scattered. They will fall together again. Because of the increase in mass-pairing by the splitting up of the original bodies, and also by the loss of energy in their fracture, the average intensity of the fragment-pairs must be less than that of the original pair; and since the bodies were led to collide, the fragments will do likewise, to a partial degree at least.

But these secondary collisions between the fragments occur under the same conditions as did the primary collision. There is to be no elasticity, and no shrouding of the energy under the mysterious term "heat." Therefore the secondary collisions can result only as did the primary, viz: in a further subdivision or comminution of the fragments. And these secondary fragments must again collide, in tertiary collisions, etc., etc.

To this process there can be but one inevitable end. Collision, fracture and disgregation must take place again and again, although with diminishing violence, until a condition of permanently stable equilibrium is reached by *the fragments becoming so small that they no longer collide.* Instead, they will have come, one after another, as each became small enough, to

adopt elliptic or hyperbolic orbits of revolution about on another, without collision, in permanently stable equilibrium an with energy perfectly conserved. Their subpermanent energ will have become of the "permanent" type. Their inelasticit will have become elastic, not by some miraculous metamorphosi from ordinary matter into molecular matter, but merely b foregoing contact at all, procuring reversal of motion by force action "at a distance," instead of by collision supposed, i violence of all natural experience, to be perfectly elastic.

But this permanency of energetic condition is just what i called "heat," viz: a permanent mode of motion-energy an space-energy between the particles of a body, resultant fron inelastic collision. The prime characteristic of heat is its per manence. All other forms of energy apparently tend to tur into heat, pretty completely, at all times, while the heat tend to remain heat. That is why heat has been called "the waste heap of the universe," and the prediction has been freel indulged in that ultimately all other energy in the universe mus become heat.

The writer would explicitly avoid giving countenance to s extreme a belief as this. Yet undoubtedly the prime charac teristic of heat is its permanence and stability of equilibrium when considered as a result of and in contrast with the abrup instability of the mechanical energy of solids and liquids movin in contact. Temporarily, at least, the energy has reached permanence of form which must extend over a period coverin many millions of the vibrations of the very tiny mass-pair embodying the heat—far too long to permit the hypothesis o collision occurring. For there is no collision known to scienc which is not somewhat inelastic and dissipative of its energy.

The attempt to define heat has therefore accomplished it first stage. *Heat is a mode of motion and of separation amon a swarm of tiny fragments of the mass of the hot body, non of which possess subpermanent orbits.*

As to superpermanency of orbit, discussion of the possibilit of that being a part of thermal interaction must be deferre Other questions as to the form of these intermolecular orbit and interactions must also be deferred, until the many divers peculiarities of heat may have been gotten more clearly in view To this end good use can be made of the thermal diagran

And when all the data are thus displayed it may appear that several of the other sources of heat, which are much more obscure in form and nature than impact and friction, have characteristics so like to these that this entrance into the thermal ant-hill through only one of the many doors may not seem so one-sided and inconclusive a plan after all.

CHAPTER VIII.

THE THERMAL DIAGRAM.

Let heat-producing impact and friction be imagined as occurring against a specific weight, such as one pound, of some familiar solid, such as ice. Let it be imagined that the ice were originally at the very lowest temperature of which scientific investigation has had experience, where the ice would be a very cold, hard, brittle solid. The result of the impact and friction would be to raise the temperature of this solid; and if it were continued sufficiently, it would ultimately melt the ice and carry the resultant water through all the thermal experiences of which the substance H_2O is capable.

It will be of convenience to represent this process graphically. And since the previous analysis has shown energy always to consist of the product of two independent variables, it will be most natural to regard thermal energy also, from the start, as made up of the arithmetical product of two variables. Indeed, if heat is to be considered as a "mode of motion," or one form of mechanical energy, at all, it *must* be considered as the arithmetical product of two variable factors; for this constitution was everywhere found to be a prime characteristic of mechanical energy.

When any quantity thus consists of the product of two variables, it is most conveniently represented as an *area,* upon a field of rectangular coördinates. The two independent factors then become the two coördinates, respectively. But if heat is to be depicted thus, the identity of the two coördinate factors stands, at the start, as a matter of guess-work. An easy first guess for one of them is *temperature;* for temperature has been, from the beginning of thermal science, recognized as a prime factor in heat. And yet it has also long been known that temperature is not heat.

If this first guess has been a true one, in accord with the natural facts, then the second coördinate (regarding which nothing can be known at the start) will prove to be identical with

some natural prime factor in thermal phenomena also. But if the first guess should prove to have been wrong, then the whole graphical situation will fall into chaos, in a *reductio ad absurdum.*

But if a fair knowledge of thermodynamics on the part of the reader be assumed, it will be plain that the selection of temperature as one of the prime factors of heat is no wild, irresponsible guess. It is now more than eighty years since Carnot proved conclusively that temperature was the one fundamental feature of heat in the guidance of work-performance; and that, too, by pure empiricism, without attempting any definition of either heat or temperature. It is now more than a quarter of a century since Lord Kelvin defined the only true temperature-scale in terms of *work,* rather than of heat; and since Maxwell proved, in the mathematical theory of the so-called "perfect gas," that temperature was the translational kinetic energy of the flying particles of thermal matter, and therefore a real physical quantity.

But the writer wishes especially to avoid humbugging both himself and the reader by starting from premises which are laid down too rigidly, as if they were absolute law. For in this whole field of discussion we possess no such rigid premises—unless the laws of Kepler and Newton, which are now unquestionable, be such. The ideas as to temperature, heat, entropy, etc., must fit the facts; that is all. While the present discussion has started rigidly enough from the exact mechanics of Kepler, Newton and La Place, because their work has stood the tests of centuries, yet the entire hypothesis that heat is "a mode of motion" at all, it must be remembered, yet hangs unsettled in mid-air. If it can be made to appear that what exact data we possess as to heat fit what exact data we possess as to mechanics, heat may be accepted as a mode of motion. But until that is settled our premises must remain assumptions and guess-work, and should be defined clearly as such.

With this to start with, the thermal diagram may be put under construction, as at B, at the lower left-hand corner of Fig. 8. The vertical axis of this diagram is to measure "absolute" temperature, along the axis OT. This locates two horizontal axes: one at XX for the absolute zero of temperature, and the other at ZZ for the Fahrenheit zero.

Areas, then, are to measure heat, and heat as supplied by

impact and friction. But the other coördinate of the diagram is for the present unknown. Therefore it must be defined in terms of the two quantities which are known. Plainly, it must be the result of dividing area (or heat) by height (or temperature).

But in doing this it must be remembered, as was stated in the opening pages of the First Paper, that energy is a name for a *change* of condition only, and not for something absolute. Therefore, since conditions change constantly with increments of energy, our definition must be confined to exceedingly small increments of energy at a time; or, in short, must be stated as a differential.

Let the horizontal coördinate be given the symbol N. Then our stated premises are defined mathematically by Equation 29,

$$dN = \frac{dQ}{T} = K\frac{dT}{T} \tag{29}$$

wherein dQ signifies the quantity of impactive energy absorbed, T the absolute temperature of the body at the time of impact, and K the specific heat of the body.*

Starting therefore at B, Fig. 8, the curve which represents the thermal experiences of the ice under impact must be some such an one as BC, simultaneously rising in temperature and departing to the right, with positive values for dN. The equation for this curve can be had by integrating Equation 29, which is easily done if K the specific heat be a constant. The result then is

$$N - N_0 = K \log_e \frac{T}{T_0} \tag{30}$$

wherein the zero-subscripts refer to any original, and the unmarked symbols to any final, condition. Should the specific heat be not a constant, the form of the curve would be slightly altered

*The writer has no desire to impose upon the reader the suggestion, from the above language, that the thermal diagram thus developed, which will prove to be identical with the well known entropy-temperature diagram, is originated in these pages. But the ultimate significance of this diagram has been so doubtful and obscure that the writer wishes to avoid its adoption in the premises. The argument has been arranged to be self-contained. No dogmatic assertions have been made in the premises, except the mechanics of Kepler and Newton and the principles of the conservation of mass and energy. If the conclusions appear to be too faulty, or too extensive to be easily accepted, the blame will not be lost amid a haze of ill-defined or too rigidly assumed premises.

and the integration not so easy; but the general conclusions would be unaltered.

During this warming of the ice by impact there occurs, apparently, no change in its physical state. The ice remains a crystalline solid. Only its temperature alters, together with the yet undefined horizontal factor of heat. From this fact the process and curve BC, with Equation 30, have been given the name *isomorphic*—the syllable "morph" indicating *form* and the prefix "iso" the fact that it remains unchanged. The term is used in contrast with *metamorphic,* which is applied to those processes, such as fusion, vaporization, etc., where the physical state of the body does undergo a material change in form.

From Equation 30 it is clear that if the original temperature T_0 from which the ice was warmed be imagined as occurring at lower and lower points, the point B must be regarded as moving indefinitely to the left, as it approaches the "absolute zero" limit of temperature. The curve BC must be asymptotic to this axis, as it trends to the left. No finite value of $N - N_0$ can be great enough to make $T_0 = 0$, when the value of T is finite. That is to say, the absolute zero of temperature is absolutely unattainable in nature. It is as unreal as is any absolute zero of either space, motion, force or mechanical energy.

As the isomorphic BC proceeds to the right, however, it finds an abrupt termination at the point C. When a temperature of 32° F. is reached the ice begins to melt. Further supplies of impactive or frictional energy continue to be absorbed, and it is to be inferred that they take the form of heat. This fact can be easily proven by other experiments, such as mixing the ice with red-hot iron, when the iron will be cooled and the ice melted. But the latter would not be warmed. It would absorb the heat given up by the iron as "latent" heat, in a change of physical state at constant temperature.

Such a process would be shown in Fig. 8 by the straight horizontal line CA. It is horizontal because the temperature does not change during the heat-absorption. It is straight and dotted because it does not represent a continuous process, but a gap, across which jumps molecule after molecule of ice, as it acquires heat enough, in unstable equilibrium.

It is known that the amount of heat absorbed in this process is large (142 B.t.u. per pound). It is known that the melting

consists of a breaking up of the ice-crystals into formless liquid. The volume of resultant water is less than that of the ice; nevertheless, there is reason to believe that the space actually occupied by the substance itself increases during fusion. The ice-crystals may be likened to a cord of fire-wood; when converted into saw-dust less space will be occupied than by the pile of logs, with their many interstices; yet if these interstices be deducted from the original volume, the space occupied by the wood has increased. A few large interstices have been exchanged for many small ones—the former being distinguishable from the substance, but the latter not.

Such a process as this melting of the ice is called *metamorphic,* implying *change of form,* in contrast with the isomorphic processes such as BC. The metamorphic processes are all virtually isothermal. The heat which they absorb is called *latent,* because imperceptible by the thermometer, in contrast with the *sensible* heat absorbed in the isomorphic processes.

Equation 29 easily becomes

$$dQ = T\,dN \tag{31}$$

If the relation between T and dN be known, as by knowing the specific heat, this equation can be integrated into an expression for $Q - Q_0$, the quantity of heat involved in passing from any one condition to any other. The quantity dQ is shown in Fig. 8, as a very narrow vertical rectangle, having a height T and a width dN. The integration of this rectangle will develop the area beneath the curve bounding the upper ends and limited at right and left by the ordinates of original and final conditions. Thus, the heat required to warm the ice from the condition B to the condition C will be given by the area BCc. That required to melt the ice will be given by the area CAOc.

The melted ice at the point A constitutes the arbitrary zero of our steam-tables. If the process of adding energy by impact and friction be continued, the water will renew its rise in temperature, this time along the isomorphic ADD'W. At the same time its vapor-tension, or the pressure which is exerted at all temperatures by the vapor struggling to free itself from the water, increases.

If this vapor-tension should happen to equal the pressure exerted upon the water by the surrounding objects by the time

the temperature D is reached, the molecular equilibrium again becomes unstable. The water cannot absorb more energy in the shape of internal motion without its internal centrifugal pressure exceeding the external centripetal pressure; wherefore it must burst. So burst it does, molecule after molecule, into steam, as fast as each molecule becomes hot enough; just as pop-corns do in a corn-popper.

The popping of each molecule alters its condition suddenly and completely from that shown at D to that shown at E. There is no stability of equilibrium between D and E, and no molecule may stop part way after having once started across. If the thermal condition of a quantity of water and steam is ever shown at any intermediate point, such as P or Y, it means not that the entire pound of molecules is in such intermediate condition, but that one-half or less are in the condition D and the other half or more are in the condition E.

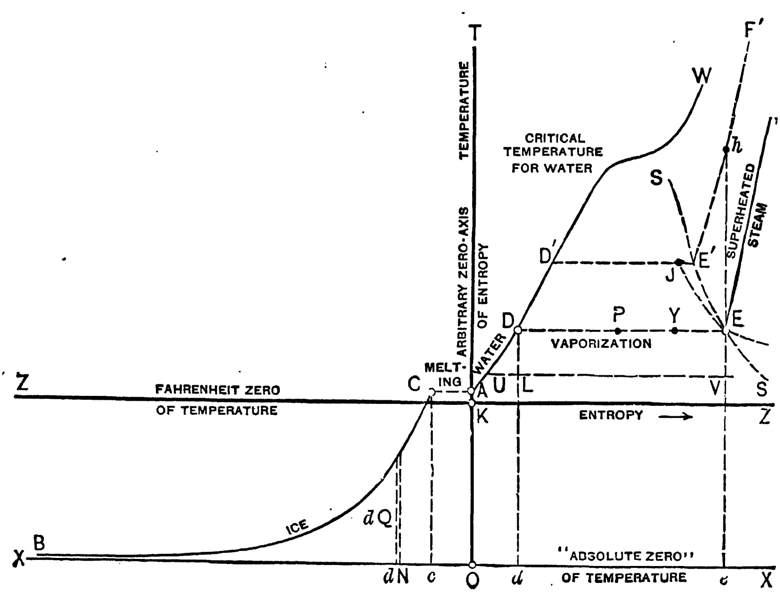

FIG. 8A.

The latent heat of vaporization is shown by the area beneath DE, or dDEe. It is much greater than the latent heat of fusion, as shown by comparing this area with CAOc. In this case there is no doubt as to the increase in volume. The volume of each molecule after "popping" is several hundred times that before, the ratio depending upon the pressure under which vaporization occurs.

Indeed, this increase in volume is so great that it expands into visibility an energy-quantity which has hitherto been negligible. This is the *external work*. The energy supplied to each molecule along DE consists not only of that required to burst

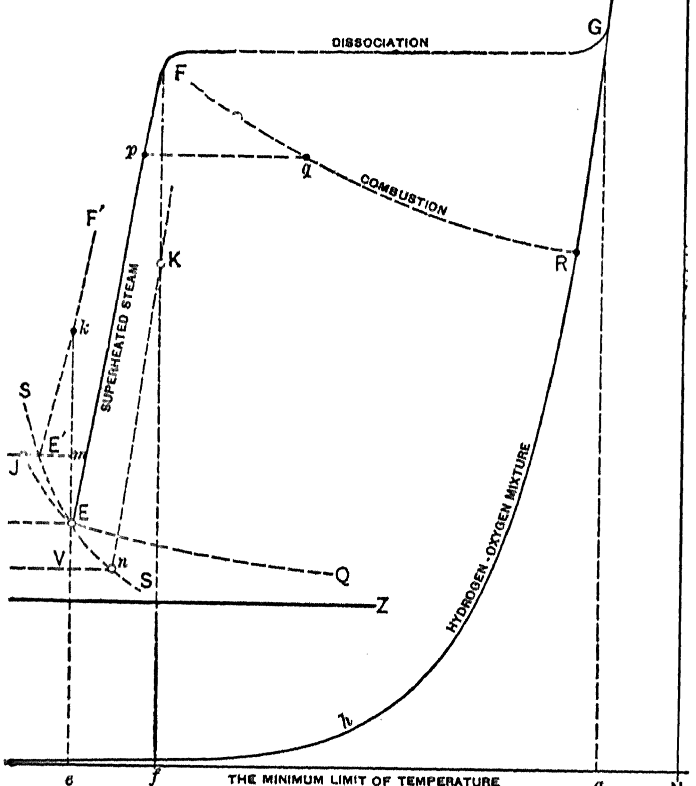

FIG. 8B.

the molecule against its own internal bonds of unity, called the *disgregation-work,** but the external forces must be pushed back also, throughout a considerable increase in volume. Yet in this case the external work amounts only to about one-ninth to one-fifteenth of the disgregation-work, so powerful are the congregative molecular forces.

If the pressure upon the heated water had been higher than that permitting vaporization at D, that process would have been delayed until some higher temperature had been reached, as at D′. Therefore, there may be as many different vaporization-levels as there are different pressures. The ends of these various metamorphic lines, such as DE, D′E′, etc., form a curve SS which is called the saturation-curve. But the student is especially warned against thinking of the saturation-curve as representing a process, as do most of the other curves in the thermal diagram. *There is no process known to the boiler or engine rooms which will convert saturated steam of one pressure and temperature into saturated steam of another pressure and temperature.* It takes a combination of at least two processes to do this, and an impossibly delicate balance of the two, at that—unless the steam be in contact with water with which it may interchange heat promptly, in which case the heat is added not merely to the steam, but to the water also.†

If the addition of energy by friction and impact to the pound of H_2O at E be continued still further, the steam will rise in temperature again along the isomorphic EF of superheated steam. The volume increases almost proportionally with the temperature. The disgregation-work has now become the minor portion of the energy absorbed, the external work being in ascendancy.

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*"Disgregation" implies the scattering of a flock, the opposite of "congregation," the gathering of a flock.

†For this reason the use of the term "specific heat of saturated steam," meaning the difference in the total heats of two points on the saturation-curve separated vertically by one degree of temperature, cannot be too strongly condemned, as loose and misleading. The term "specific heat" is used generally and properly to signify the quantity of heat which, added to a substance, will raise its temperature by one degree, the physical state remaining constant. The use of the term "specific heat of saturated steam" therefore leads the student to infer, most naturally, that if you add to saturated steam containing the total heat H_1 the heat $H_2—H_1$, wherein H_2 is the total heat of saturated steam one degree hotter, there would result saturated steam of the total heat H_2 and temperature T_1+1. Yet nothing could be further from the truth.

At some higher temperature, such as F, there arises a new condition of unstable internal equilibrium within the molecule. If energy continues to be added, molecule after molecule of the steam bursts again—but this time not into a new "physical" state of H_2O, but into a new "chemical" state of dissociated hydrogen and oxygen. The energy now absorbed in potential form, along the metamorph FG, is some seven times as great as that absorbed in vaporization; just as that absorbed in vaporization was some seven times that involved in fusion. Moreover, it is not called latent thermal energy, but *chemical* energy. For this practise there are excellent reasons; but it is one of the offices of Fig. 8 to show plainly how much more closely allied are heat and chemical energy than is commonly supposed. It is also to remind the reader that chemical energy is distinctly a latent form of energy.

The increase in volume which is involved in this dissociation amounts to only fifty per cent.; and the external work involved amounts to less than two per cent. of the whole. Therefore it may be said that virtually all of the energy absorbed goes into latent, chemical or disgregative form.

Should the process of adding energy by impact—which has now become most difficult, because of the almost perfect elasticity of the substance—be persisted in, the temperature again rises along an isomorph, GH; and to this isomorph there is no further interruption, by instability of internal equilibrium, so far as the writer is aware. Moreover, because of the high temperature already attained, the horizontal departure dN involved in the absorption of a thermal unit dQ has become so small, and the corresponding rise in temperature has become so great (because the specific heat is much less than for liquids or solids), that the isomorph GH rapidly becomes approximately asymptotic to some limiting vertical axis, not yet exactly located.

Should the oxy-hydrogen mixture which is illustrated as active in the isomorph GH be cooled again, by the conduction of its heat to colder bodies, it may be said (though the statement is unsupported by what has preceded) that it will return along the curve HGFEDACB which it came up. But if the two gases be separated, and then cooled, their thermal condition will be shown by the curve HGRh.

If, at any comparatively low temperature such as *h,* the

gases be mixed again and then heated, when they reach their temperature of ignition, as at R, they will again combine, releasing chemical energy which must be absorbed thermally. In order that the combination may be complete, the thermal energy must be absorbed from the substance, by some outside body, bringing the mixture again into the condition F. The sum of these processes may be illustrated by the curve RF, although an intermediate passage of some portions of the substance into the condition H is involved.

In the curve hRH is visible a smooth transit of a substance from one thermal condition which is asymptotic to the axis of absolute zero of temperature, to another thermal condition which is asymptotic to an axis at right angles thereto. In the curve BCADEFGH is visible a somewhat similar path of thermal transit, broken by localities of unstable equilibrium, it is true, but terminating in each direction in a smooth curve of stable equilibrium, which is asymptotic to horizontal and vertical axes, respectively. The field visible between these curves is crossed, from curve to curve, back and forth, by the familiar processes of melting, freezing, vaporizing, condensing, combustion and dissociation. The curves themselves represent the even more familiar processes of warming and cooling.

If it be remembered that every substance known, except a few gases rare even in the chemical laboratory, occurs in all of the three physical states: solid, liquid and gaseous, and is subject to chemical dissociation and combination, in endothermic and exothermic processes; and if it be remembered that all these processes, for all these substances, might be represented upon Fig. 8, with no departure from what is already there, except in the number and confusion of lines and in the choice of convenient scales; and if it be further remembered that all the processes for conversions between heat and work, as well as those between heat and chemical energy, may be displayed upon this diagram (though as yet there has been no reason to refer to some of them)—it becomes evident that Fig. 8 places before the eye in comprehensive form a pretty complete picture of the data which are concerned in and essential to a complete understanding of the relationships existing between heat, work and chemical energy. If there be any truth in the hypothesis that heat is indeed a mode of motion, and chemical energy a mode

of arrangement, it should certainly come out from a thorough inspection of Fig. 8, in terms of the analysis of mechanical energy already accomplished.

This task must be deferred for later papers. For the present it is desired merely to call attention to the general fact that the curves of energetic equilibrium displayed in Fig. 8 have the same general form as those displayed in the Sixth Paper, covering all known cases in purely mechanical energy. That is to say, they vary, on either side of a central, or mean energetic, condition, which central condition is not definitely located from any rigid or absolute base, indefinitely toward and along two axes (at right angles with each other) to which they become asymptotic and which *they can never reach.* Unlimited departure *from* either axis is natural and imaginable; but unlimited approach *toward* either one is not. While it is true that the upper limb of the heat-curve of Fig. 8 is not a true asymptote, as it is there drawn, yet its similarity to one is obvious. Whether or not it be a true asymptote will be discussed more in detail later.

CHAPTER IX.

Mechanical Concepts of Thermal Phenomena.

A. Pressure and Volume.

If it be assumed that the preceding papers have supplied complete data for the understanding of work, heat and chemical energy, in so far as the last named is related to the other two, the first task is to construct therefrom mechanical concepts of the four fundamental attributes of matter which are active in these fields, viz:

(a) Pressure,
(b) Volume,
(c) Temperature, and
(d) Entropy.

These are all thermal attributes. Pressure and volume are thermodynamic in character, bridging the gap between heat and work. Temperature and entropy are almost purely thermal attributes, but also are active in thermochemical phenomena.

Besides the above attributes, it is also necessary to explain in mechanical terms, two fundamental processes, viz:—

(e) Heat-development and transfer, and
(f) Thermodynamic work-performance or work-absorption.

Of all of these phenomena, according to the writer's view, the one easiest to comprehend is volume, and the hardest one is pressure. Both are inextricably mixed up with heat-action; and yet the occurrence of mechanical action where pressure and volume are involved, yet where no heat-changes are perceptible, is common. But in all cases, if the facts be closely examined, both pressure and volume will be found to be thermal phenomena. For no substance can be imagined as reduced to a zero of either pressure or volume without first being carried to that unattainable thermal condition, the absolute zero of temperature.

Volume. If the idea that heat is a mode of motion is to be ıered to consistently, the volume exhibited by any body must the effect of the separation between its component mass-ticles. Since the internal condition of the body is permanent l stable, these mass-particles must be in a free condition of ural equilibrium. If so, their relative separation, in the face their mutual attraction toward each other, as well as in the e of external pressure, must be maintained by motion of olution about each other. This is the only possible me-nical explanation of the occupancy of space by elastic matter.

In order to explain volume alone, this internal motion might regarded as purely circular in form; and this is the simplest a with which to start. In that case, the centrifugal and ıtripetal forces would be exactly balanced, and there would no active exertion of expansive pressure outwardly. There uld be, however, a passive, and at times a stalwart, resistance compressive pressures acting radially inwardly.

Since the motion of the particles (under the above assump-n) is purely tangential, its velocity must increase as the ume of the body becomes smaller. But during any such a inge, where circularity of motion is conserved, the energy nnot be conserved. Energy must be abstracted in order that volume of the body should become smaller and the tan-ıtial velocities greater.

The only special provision to be made, in imagining the ume of a body as made up of a vast number of tiny mass-irs, revolving as described in the earlier papers upon me-ınical energy, is that these many orbits must lie in all sorts planes, interacting at oblique angles, whereas the elementary its studied were all specified as confined to a single plane. t this provision introduces no new principles of action.

Pressure. The above hypothesis furnishes no explanation pressure active outwardly. For this can be explained only on the assumption of eccentricity of orbit between the par-les, involving as it must a lack of balance between centrifugal l centripetal forces at both apastron and periastron. Indeed, s lack of balance is true of every point of an eccentric orbit cept the mean energetic point; and even there, although the rces are balanced, there exists an unbalanced fund of radial

motion, acting outwardly on one side of the orbit and inwardly on the other.

Now all known conditions of matter exhibit pressure or force per unit of area. This pressure is of the two sorts already mentioned, viz: first, the passive resistance to external forces which is exhibited at its best in the solids; and, secondly, the spontaneous expansive pressure which is best exhibited in the gases. But all known forms of matter exhibit both of these phenomena. There are no known solids so dense and hard that they do not exert some slight expansive vapor-pressure, although the passively resistant form of pressure is overwhelmingly more prominent. On the other hand, there are no known gases so diffuse that they do not exhibit some slight resistance to deformation, called "viscosity," such as is familiar in all liquids and solids, although their gaseous characteristics are overwhelmingly more prominent.

Therefore, since both sorts of pressure are to be found in finite degree in all cases, and since neither sort of pressure can be explained mechanically without finite eccentricity of orbit, it must be assumed that all molecular orbits are somewhat eccentric. Neither circular nor rectilinear orbits are possible.

This supposition agrees, too, with the mechanical principles developed in the Third Paper: That eccentricity of orbit could be removed only by radial action, and therefore that, as the eccentricity decreased and the radial phenomena became less and less, the difficulty of further reducing the eccentricity became greater and greater; so that it is unimaginable that eccentricities should ever be reduced to zero, by activities depending upon the eccentricity for their effectiveness. Zeros of pressure and zeros of eccentricity of orbit must be alike dismissed from consideration, as conditions impossible of occurrence in nature, constituting limits which may be approached but never reached.

The same is true of infinity of eccentricity, or zero of curvature, of orbit. Radial departure between two bodies can be created only by tangential action at periastron, as in the illustration of the cannon-ball. Therefore some radial component must always be retained. It is impossible to imagine tangentially imparted intensity of radial motion ever getting to the point where there was no tangentiality; or where, in other words, the orbit had ceased to be a hyperbola and had become a straight line.

Now, of the two sorts of pressure just mentioned, it is plain that outwardly active, or *expansive,* pressure can be explained only by orbits possessing eccentricities greater than unity. Only in such cases would either member of a mass-pair be able to free itself from its mate or mates, and fly outwardly until arrested by external resistances. *Passively resistant* pressure, however, which absorbs energy in forceful resistance to deformation, but which makes no effort to expand beyond fairly fixed limits, can be explained only in terms of orbits having eccentricities below unity; that is, elliptic orbits. For all such orbits contain within themselves a fixed outward limit of motion, beyond which there will be no trespass. But to any arbitrary limitation of motion *within* those limits, by forces exerted from without, to the shortening of the natural length of the ellipse, there would be exerted stout resistance.

Since both sorts of pressure are found in all natural conditions of matter, it is necessary to assume that in all molecular energy-systems there exist both elliptic and hyperbolic orbits. Since the passive form of pressure is much greater than the expansive sort in solids, it is necessary to assume that in solid matter the far greater portion of molecular mass is revolving in elliptic orbit, only minor fragments following hyperbolic orbits. In gaseous matter, on the other hand, it is necessary to assume that the major portion of the mass moves in hyperbolic orbit, only a minor portion retaining elliptic motion.

It is only in the unattainable, limiting case, however, that all of the mass could assume hyperbolic orbits; for it is only by action which depends upon mass in elliptic orbit that any other mass may be given a superpermanent intensity of energy in hyperbolic motion. This fact makes it clear that no combination of natural circumstances could ever develop in matter a state where all elliptic motion had ceased and all the attributes of a solid had disappeared. And this fact, too, agrees with all the observations heretofore drawn, viz: That matter and energy depart, in either direction, from a central condition where all things are balanced, toward extremes where one or the other condition becomes exaggerated or suppressed, only with steadily increasing difficulty; and that no imaginable conditions or processes could ever force things to the point where any of these normal attributes of matter had become zero. But such an

impossible state of affairs as matter in which all orbits were hyperbolic and none elliptic would constitute, if it could ever be attained, the much talked of "perfect gas." Therefore the "perfect" gas does not and cannot exist. Reliable scientific authority does not teach that it does or could; but the general concept of the perfect gas has been used so irresponsibly, and with such mischievous results, by many teachers of engineering thermodynamics, that we shall refer again to its absurdity as a concept of natural matter.

To return now to the concept of expansive pressure as a manifestation of hyperbolic motion: Such a concept of pressure as a bombardment of the walls surrounding a hot substance by a multitude of radiating molecules is by no means new. The trouble with it is that it doesn't explain. The trouble is not that such a bombardment could not exert the pressure. The trouble is that the pressure is exerted continuously, without loss; whereas every bombardment known to human experience involves several losses. All the projectiles are lost. All their energy is lost. And usually the wall itself is also lost.

It is of no use, in this juncture, to have it explained to us that the wall is perfectly elastic and the projectiles are perfectly elastic, and that both wall and projectiles are indestructible. We know nothing about any such things. The bombardment-explanation of pressure has been to the author, ever since he first heard it as a student, a blind failure to explain the obscure, because attempted with the aid of something still more obscure. And, so far as he can discover, it has been equally so to every sincere student.

Another aid, and also obstacle, to the comprehension of pressure lies in its similarity to and its contrast with temperature. If we are to rely upon the linear kinetic energy of the outwardly flying particles having hyperbolic orbits to explain pressure, what is left to explain temperature? Moreover, temperature has already been defined as this linear kinetic energy. For pressure and temperature, while sufficiently alike in some respects to be considered identical, are yet strongly contrasted in some other respects.

Speaking broadly, the active expansive pressure of the vapors and gases is roughly proportional to temperature. Similarly, the

passive resistant pressure of the solids is inversely proportional to temperature. It is the higher temperatures which create vapors and gases, with their great expansive and slight resistant pressure. It is the lower temperatures which develop solids, exhibiting the reverse of this. In the more permanent gases the proportionality of expansive pressure to temperature is almost exact, according to Boyle's law, while the viscosity, or passively resistant pressure, or "solidity" as we might truly call it, is almost inversely proportional. In the liquids the vapor-tension varies directly and widely, though not exactly proportionally, with the temperature. In the solids the connection between expansive pressure and temperature is much more obscure, but it is roughly visible and it is nowhere reversed.

All of this evidence, if it were not for the bombardment difficulty, would fall in excellently with what was said as to elliptic and hyperbolic motions. The almost circular, or low-eccentricity, motion of the "solid" molecules is fitted as naturally to explain the stout passive resistance of the solids as the highly eccentric hyperbolic motion of the "gaseous" particles is to explain active, expansive pressure. For a given mass can most effectively resist a deflecting force (without itself undergoing transformation) when it is moving at a high velocity *normally* to that force; and a circle is normal to every line approaching its center from without. The hydraulic jets of the old-fashioned placer-mines, in California, for instance, were said to come from their nozzles with such a velocity that a man could not strike an ax into the water. Therefore it is quite reasonable to imagine the solid state of matter as consisting of mass-particles situated very close together, yet kept apart by a very high velocity of almost circular motion about each other; for in such case the velocity must increase as the particles come more closely together, or in other words, the hardness and rigidity of passive resistance to external pressure must increase with the density—which is just what is observed in nature.

Such a system would be elastic, but non-expansive. The abstraction of energy would increase both density and hardness of resistant pressure, while the addition of energy would soften and expand it into a liquid or a gas. Everything about the explanation would be beautifully consistent, *if* only some sufficient substitute for the bombardment could be found, to explain the

method of application of these external forces and energies to the tiny revolving systems.

The key to this situation, as also to the contrast between pressure and temperature, lies in the following facts. *When pressure performs work, there is no transfer of energy across the gap where the pressure is felt. Whenever and wherever temperature is felt, however, there does occur a transfer of energy across the gap.*

The first of these statements appears at first sight paradoxical. Nevertheless it is true. When pressure performs work it is the surface bounding the energy which moves, not the energy across the surface.

To explain, when pressure performs work the energy is most commonly what the writer calls *transient* energy. In transient energy the true source of energy is some more or less distant driver, from which the energy comes by some carrier which is itself a mere inert "pusher," exerting the pressure which is under discussion. In such cases the amount of energy transmitted is quite independent of the mass of the carrier. Instances of transient energy are found in the fluids supplied to hydraulic cranes and pneumatic tools, where a remote pump or air-compressor furnishes the energy, which the water or air under pressure merely transmits to the driven piston.

The same is true of the steam of a steam-engine, during admission. Then the energy is furnished solely by hot-water expansion in thc boiler; the steam in the steam-pipe and cylinder undergoes no energetic change whatever, but serves merely as a fairly frictionless transmitter, neither expanding nor contracting, nor undergoing acceleration or retardation. It is only after cut-off that the steam itself becomes an active source of energy.

It is transient energy, too, which is active in the water of a penstock. It is only within the guide-blades and wheel that the water's own energy becomes active. It is transient energy which gives to the greater number of machine-parts their value. All rods, shafts, chains, belts, etc., are busy handling transient energy. The only pieces of metal in which we utilize the energy stored in the metal itself are weights, springs and hammer-heads.

In all these cases it is obvious that it is the surface bounding the energy, the surface where the pressure is felt, which moves as the energy is transmitted, and not the energy across the

surface. In all cases of transient energy the energy-fund of each piece remains constant. It is the piece itself which moves.

In the case of non-transient energy, such as weights, springs, hammers or bodies of expanding steam or gas, where the energy-fund of the body does change, it is still true that no energy is transmitted by pressure across the surface of contact. *It is only as that surface moves* that energy is, or can be, transmitted. The fact that the energy is developed within the body, rather than transmitted through it, does not alter the case. A weight resting at constant height, a spring under constant distortion, a hammer-head which hits no anvil, or a body of steam under constant volume—none of them transmit any energy, although they may exert great pressure.

It is the essential characteristic of expansive pressure, then, if we are to follow the mechanical hypothesis strictly, that it consists of a bombardment—because it is associated only with the hyperbolic paths of particles which would not turn back except for the external pressure—*but of a bombardment in which no energy is transferred.*

Now such a bombardment is very different from anything known in military experience. Such a bombardment would require the party bombarded to kindly catch all projectiles, without impact or friction, and to return them with velocity conserved and direction of motion reversed. Such an action has been provided, in the explanations of pressure commonly given, by the assumption of "perfect elasticity." But the reversal of direction of motion with perfect conservation of energy is only found in nature, it has repeatedly been pointed out, in the mutual revolution of mass-portions *around* one another. Therefore, if the hypothesis that heat is a mode of motion is to remain intact, pressure must be regarded as the result of the radiation from the surface in question of a continuous flow of tiny projectiles; but these projectiles, instead of actually striking and rebounding from contact with the particles of the other body (for pressure can be felt only *between* two bodies), must be imagined as being met half-way by a similar swarm of projectiles radiated from the other body, as revolving about them in a "swing-opposite-partners" fashion, returning home with pressure exchanged but energy conserved.

According to this, there can never occur what is commonly

called "contact" between mass-portions. All action is at a distance. Contact, so-called, is merely the approach of two systems of particles into such propinquity that they begin to feel—or rather, to manifest perceptibly to our crude senses—the repulsive effect of the bombardment of projectiles from the other body. In gases and vapors these projectiles travel far and embody considerable mass; most of the mass of the body is in projectile form; therefore the repulsive effect is far reaching. In solids, on the other hand, the projectiles are few and of short range. The major portion of the mass of each molecule is engaged in elliptic motion, exerting no expansive pressure. Such a central and self-contained portion of the solid molecule is called the *nucleus,* in contrast with the projectiles which it sends out upon hyperbolic orbits. In the case of solids the expansive pressure of these projectiles can scarcely be felt. But as soon as the external pressure drives these feeble projectiles home upon the rapidly revolving (and therefore rigid) nucleus, the resistance of the latter promptly becomes considerable.

An excellent simile for this situation is the exchange of pressure between bodies of troops in war. There the sense of contact is exchanged by means of flying projectiles, or bullets. Each body experiences a force of repulsion from the other by means of these projectiles, long before the two bodies come visibly into contact. An observer of military action from some commanding height, who was familiar with primitive warfare but knew nothing of guns and bullets, would be much surprised and puzzled in seeing two orderly masses of men, moving in opposing directions with antagonistic aims, slow down as they approached within a considerable distance of each other, hesitate, waver, desist from their predetermined purpose, break into small and irregular detachments at the points of closest approach, take on lateral motions of various sorts, and finally the smaller body reverse its direction of motion in response to the repulsion of the larger, and depart with a velocity higher than that of approach—all of which is quite similar to what we observe in the "contact" between solid portions of matter. The imaginary observer of all this could not see the flying bullets which were the active agents in this transfer of pressure from army to army, and he would be entirely at a loss to explain it, no matter how wide had been his experience with warfare conducted with swords,

spears and chariots—until some one mentioned the bullets.

Now the impulse carried by the bullets is a moral one, rather than physical, and is dependent as much upon the accuracy and timeliness of their discharge as upon their mass and velocity. So this simile is truly a simile, rather than an accurate scientific analogy. Yet it may be extended usefully to the contrasts between gases and solids. An army progressing freely through territory where opposition is not imminent, divided into many separate bodies, with extended columns, flying detachments of cavalry, scouting and foraging parties, skirmish-lines, etc., may be likened to a gas. It will occupy all the space it can.

Such an army, meeting another such, would feel the pressure of contact very gradually. A skirmish-line would be driven back here or there; scouting parties and flying detachments would become more careful and stay nearer home. But only after appreciable time would these outlying features be driven into consolidation with the main body, and the entire mass be compressed into a form so dense that pressure felt at any one point would be transmitted with promptness throughout the entire mass.

The armies thus condensed, as in active conflict in difficult country, where the enemy's pulse is not easily felt, might be likened to solids. In such case there would be little premonition of contact, until it occurred with considerable force. Then there would be little elasticity. Instead of repulsion in visible reverse-motion or diversion of path, the exchange of energy would be sharp and sudden, and would occur with great loss. Contact would become synonymous with impact. The balance of power would be established through grinding wear, with the partial destruction of the reacting bodies, in disintegration and heat of conflict, rather than with a graceful yielding to the pressure from without and a gradual accumulation of available mass against the point of contact, as is the case with elastic bodies.

In all these ways the likeness between the contact of bodies of troops and that between bodies of physical mass in motion is very great. In both cases what is called contact proves, upon careful examination, to be no contact at all. It is merely an approach into sufficient propinquity so that a *perceptible* force of repulsion is experienced. And wherever the word perceptible is used it refers as much to the thing doing the perceiving as it

does to the thing perceived. There is no end of things in nature which are imperceptible to our fairly crude human senses, when applied directly, which have been made plain either by the greater sensitiveness of inanimate instruments or by scientific analysis. And apparently the universality of the manifestation of repulsive pressure throughout the universe, between each two bodies, is one of these.

The further development of this idea of the transmission of pressure by projectiles, as the only possible explanation of thermal phenomena which is in accord with Newtonian mechanics, will be deferred for a continuation of this chapter.

CHAPTER X.

The Mechanical Concept of Pressure *(Continued)*.

The mental picture which must be formed of the average molecule, in order to prosecute the mechanical analysis of heat, is as complex and varied as the aspect of an army-corps in active campaign—to complete the simile which was introduced in the preceding paper. It must itself consist of many, very many separate portions.

The major portion of this mass, in the solids and liquids, at any rate, will be devoted to the formation of a central body which may be called the *nucleus*. This nucleus is itself complex in structure, formed of many parts, which may separate, upon occasion, in independent action. Its internal motions are more or less mobile and fluid. Yet it embodies a greater degree of consistency, and approaches more nearly to the condition of a solid, than the more out-lying portions. In the mechanical concept of heat this nucleus must be conceived as composed of particles revolving in elliptic orbits, many of them of an eccentricity approaching zero. In this way the nucleus may be understood to possess a unified existence of its own, of fairly permanent dimensions, until its internal equilibrium is upset and its unity broken up, by its invasion with sufficient force by some disturbing mass from without.

Surrounding this nucleus is a swarm of particles possessing hyperbolic orbits. These, to distinguish their activities from that of the nucleus, will be called the *satellites*. Nevertheless, it is not necessary to assign any other distinguishing feature to the satellites than their high eccentricity of orbit. They may be regarded as capable of becoming absorbed at any time by the nucleus, or of being formed from nuclear particles. Indeed, it is natural to assume that this process of transfer of particles, from nucleus to satellitic swarm and the reverse, as one particle or another gains or loses the intensity of energy requisite, is going on at all times. This may occur as readily as, in the

similar army-corps, men are constantly being detached from head-quarters for assignment to distant posts of duty; while other men, these duties done, are as continually returning, for re-absorption by the central body.

Nor is it necessary to imagine any sharp definition between the nuclear swarm and the satellitic swarm. The nucleus, on the one hand, may possess some members which, although moving in almost circular orbits, yet lie far out away from the main mass. Such instances occur in our own solar system, for instance, in such planets as Neptune and Uranus, which have not completed one of their "years" since the dawn of modern astronomy; or in one of the moons of Jupiter, which is so far distant from that planet that it is barely caught permanently, against the sun's attraction.

The satellitic swarm, on the other hand, may possess many members whose great eccentricity of orbit carries them as near to the center of mass at periastron as they are distant at apastron. Such particles, which would correspond to the comets of our solar system, would alternately dive into the very center of the nuclear swarm, and then depart as remotely into space as external mass-systems might permit. The instance, in our solar system, of the Great Comet of 1882 has already been mentioned. Here is a vast mass which last experienced its maximum separation from the mass-center of the solar system when Columbus was struggling into manhood. At that time the comet was at aphelion, far beyond the orbit of the most remote known planet, drifting lazily in parallel with the planetary orbits. But whereas the planets possess sufficient tangential motion to prevent them from falling toward the sun, this comet did not. It began to fall before Columbus set sail for the Indies, and for four hundred years thereafter it fell continually, with velocity increasing each second, through unknown millions of miles, toward the sun. Finally, in September, 1882, it reached its mean energetic condition, very near the sun. Within ninety minutes thereafter it had further transformed into kinetic form an equal amount of energy with that transformed during the preceding four centuries, and had reached its extreme energetic condition, at perihelion. Within a second period of ninety minutes this gigantic scale of energy-transformation had been reversed, the second mean energetic condition had been passed, and the comet was

upon another eight centuries of outward and return motion, st purely radial in character.

'et a molecular particle acting similarly to this must be ed as belonging to the "solid" nucleus, rather than to the nsive-pressure-exerting satellitic swarm, because its orbit is ic. The influence of no external mass-system is needed to n this satellite to the sun. No external mass-system would ts pressure until it had encroached upon definite dimensions ir solar system.

herefore the molecular mechanical system is to be imagined nsisting of all sizes and velocities of mass-portion, moving l sorts of orbits. Some will move in almost circular orbits nall radius and very high velocity, others in the same form rbit with large radii and low velocities. Some will move ipses of high eccentricity, with large and small mean ener- distances. Some will move in hyperbolic orbits, tending to themselves from the system except as they are returned to outside forces. Indeed, the mechanical idea of the molecule also include the constant loss of some of these smaller cles from the molecule, their mass being made up, under ge conditions, by an equal absorption of freely roving par- lost by other molecular nuclei—just as an army-corps both and gains men continually, in the form of veterans disabled e and raw recruits received, and in the form of prisoners lost, and exchanged with the enemy.

n a general way, the proportion of satellitic energy to that odied in the elliptic orbits of the nucleus is given by the ortion of "external work" visible in the substance in question. , in the case of steam, a glance at the steam-tables will that the proportion of any increment of heat going respect- to internal and external work differs widely in ice, water, rheated steam and the mixture of dissociated hydrogen and en. If we remember that the "external" work is that por- of the energy which is devoted to holding at bay the sur- ding bodies which press in upon the water, steam, etc., it ain that the work involved is that stored in the satellitic m. During the flight of each satellite its fund of energy is d in kinetic form. During the "swing-opposite-partners" rsal of motion at the far end of the route, where the pro- e comes into "contact" with the external mass, such as a

boiler-shell or engine-piston, which is being pressed back by the hot, elastic body, this kinetic energy takes the form of "propinquity," or intensity of spacial approach. Immediately thereafter the motion is reversed, toward the original nucleus, and the energy becomes kinetic again.

Therefore, the prominence of external work in the energetic action of any body is an excellent criterion of the proportion of the mass of each of its molecules which is engaged in satellitic, or hyperbolic, orbital motion, as contrasted with that which is involved in nuclear, or elliptic, orbital motion.

In general, both the nucleus and the satellites will tend to arrange themselves in a generally spherical form. While, no doubt, an individual variation of form away from the spherical could be detected in each molecule, could it be examined closely, yet the general tendency of all particles would be to dispose their departures from the center equally in all directions, and to about equal distances.

So soon as two molecules should approach within "perceptible" propinquity, the form of each of their internal orbits, and the general form of each molecule, would be altered, by the attraction of the mass of the other molecule. The situation may be illustrated by Fig. 9, wherein the several typical forms of orbits, and their perturbation by the mutual approach of the two molecular nuclei, A and B, is shown by dotted orbits marked *a, b, c* and *d.*

Many of the orbits, including most of those embodied in the nucleus, which before were almost circular, would experience, as the result of this attraction, merely a lengthening of their major axes toward the other molecule. These are shown at *a* and *b.* Other orbits, such as those shown intersecting at *c,* which before had been ellipses much elongated toward the other molecule, would find their major axes shortened at one end, the end nearer the other molecule, by the mutual attraction between the particles themselves and their mutual revolution about each other at *c,* in a little periastron of their own. Indeed, in the greater number of such cases the orbits would before have been the hyperbolic ones of satellites exerting expansive pressure outwardly; but now they are drawn down into ellipticity by their own mutual attraction. Such orbits, however, would not be geometrically perfect ellipses.

Other orbits, such as *d,* are those which, originally hyperbolic in reference to A or B alone, are now drawn down into ellipticity by the combined attraction of both, but perform alternate periastrons about first one and then the other. Nor are these orbits necessarily elliptic. They may be of the form of a figure eight, or looped in still more intricate form, when many more than two interacting molecular nuclei are involved. The entire process of interchange of pressure, indeed, may be likened to

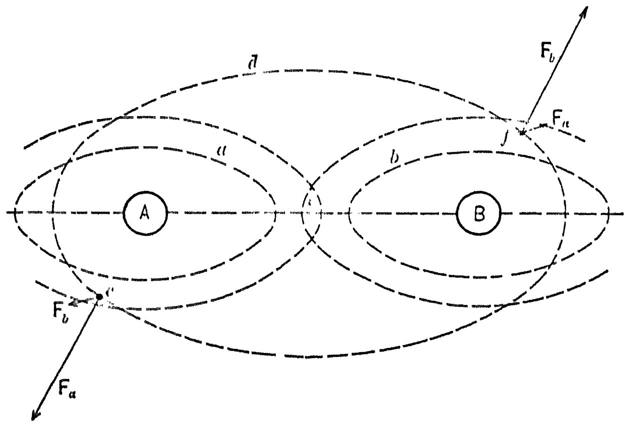

FIG. 9.

some of the figures of the old-fashioned quadrille, in which "right and left" or "ladies' chain" or "dos à dos" furnish opportunity for sturdy nuclear gentlemen to set delicate satellitic ladies flitting about orbits of the most intricate and confusing character. In which connection it is of further interest to note that all of these old dance-forms, like most savage dance-figures, arose from the primitive custom of amusement with mimic warfare, as in the joust or tournament, in which advance and retreat, skirmish and *charge en bloc,* were so arranged as to exchange pressure, but maintain equilibrium, between two opposing parties.

Any one of these satellites, at every point in its orbit, is subject to two attractive forces, one drawing it toward the nucleus A and the other toward B. In each case the attraction reacts upon the nucleus with the same force that it exerts upon the satellite. The two attractive forces, however, are seldom

equal. At any such point as *e*, for instance, the two forces would bear the ratio

$$\frac{F_a}{F_b} = (\frac{eB}{eA})^2, \qquad (31)$$

assuming that the masses of A and B are equal. But this same satellite, if its orbit were of the *d*-class (or some chance equivalent in the horde of satellites if it were not), would also be found later at the point *f*, engaged in periastron, around the other nucleus B. The point *f* being symmetrically opposite to *e*, the forces developed at *f* will be equal and opposite to those at *e*. Only now the larger force is exerted upon the nucleus B instead of upon A.

The resultants of all of these different forces acting upon the two nuclei A and B will always tend to alinement with the axis AB joining the two bodies, and will always be separative, or repulsive, in its direction. The more nearly the two bodies approach, the greater will be the proportion of the mass of each which engages in the sort of modified orbit which develops these repulsive forces.

If the two systems, each consisting of nucleus and swarm of satellites, should be generally circular or spherical in outline (according to whether confined or not to a single plane), as is our solar system, and should be of uniform density, these repulsive forces would vary inversely as the square of the distance between the nuclei. While there is no need for forcing any particular assumption as to the form of the molecule, this much is said to show that any ordinary laws of variation of force with propinquity may be explained in a sensible fashion, in terms of such a mechanical system. In general, it may be said that any two such systems, upon approach, would first be elongated in the line of their approach, which would increase their radial susceptibility, or elasticity, in that direction, and would afterward be mutually repelled, in a perfectly elastic manner, by forces increasing with some power of the propinquity.

Early in this series of papers it was shown that true elasticity can be conceived, in consistency with the Newtonian mechanics, only when the relative motion of a mass-pair is reversed by the *circumrevolution* of the two members. It now becomes clear how perfect elasticity may be manifested between two complex mass-*systems* by the circumrevolution of only a

portion of each mass-system about a similar portion of the other. The proportion of the total mass which is thus necessarily involved in penetration into and circumrevolution about the other depends, of course, only upon the intensity of the relative motion between the systems; that is to say, upon the amount of V^2 in proportion to the $M_1 + M_2$ present. It is hence easy to infer how it is that intensity of energy is a controlling factor in determining energy-transformation; for the penetration of more than a certain proportion of each system into the other would break up the internal equilibrium of both, resulting in either their amalgamation or their permanent rupture; in short, in their "transformation."

In imagining any such systems as those of Fig. 9 as constituting actual molecules, it is to be remembered that the proportion of satellites to nucleus may be widely variable. In the case of solids the nuclei must embody the greater portion of the mass of the molecule. They must be small in diameter, very dense, and very rigid from their high speed of rotation. The satellites to correspond would be few, but would possess great density and high velocity. In the case of gases, the nuclei would possess large diameters, low densities and relatively low peripheral speeds. The satellites would be relatively of much greater mass than in solids.

If the proportion of satellitic to total mass of the molecule be represented by *k*, then the repulsive force developed by two approaching systems would vary, not only inversely with some power of the distance, but proportionally to $M^2(k - k^2)$. This expression reaches its maximum when $k = \frac{1}{2}$ or when the mass is equally divided between nucleus and satellites. It is therefore to be expected that the possibilities for elastic force of repulsion should be the greatest in some intermediate stage, such as the liquid, where the satellites would bear a medium proportion to nucleus, rather than in the extreme conditions of solid or gas, where either nuclear or satellitic mass preponderates. This seems to be true in nature. In the gases the elasticity is well-nigh perfect, but the force of repulsion is small. In the solids the force of repulsion is very great, but the elasticity is quite imperfect. In the liquids, however, the force resisting compression is very great, and at the same time the elasticity is excellent.

Should any pair of mass-systems such as Fig. 9 be forced, by great energy and directness of impact, into closer propinquity than would permit the nuclei to remain intact, they will become broken up. They may then separate in fragments, or coalesce into a new nucleus, of larger dimensions and of the same or different form of arrangement. It is thus that the processes of welding solids, mixing liquids and gases, and impact and friction can be explained. Chemical combination may also be imagined as explained in the same way, although the premises must then be stated in a much more intricate manner. Obversely the accumulation within any one molecule of too much intensity of energy might easily lead to its bursting or splitting into two or more separate molecules, of the same or of different chemical nature.*

Another result imaginable from the too close collision of nucleus with nucleus would be the production of more satellites, from the fragments of the ruptured nuclei. The importance of this natural mechanical action, as an explanation of heat-formation by impact, was pointed out in the Seventh Paper, on "What is Heat?" It is now to be pointed out, in addition, that this process, like all other true energetic processes, develops a condition of stable equilibrium. That is to say, the impact of the nuclei was due to a paucity of satellites. The result of the collision is to reduce the nuclei partially to satellites, thus making good the deficit of satellites and diminishing the chances of another similar collision's occurring.

The beautifully stable balance thus established pervades all nature. It has already been referred to briefly, in our primary definition of heat. Its wide results will be taken up later at length, in the chapter upon Thermal Equilibrium (Chapter XV).

In discussing the action of these colliding systems it is hardly necessary to point out that the distance-factor, for instance, must possess a vastly different scale in mechanical, celestial and thermal energetics, respectively. Distances which constitute mean energetic ones here on the earth's surface, in the applied mechanics of engineering, would constitute prodigious extremes of separation when measured between molecules, but corresponding

*The mechanics of this hypothesis has been beautifully developed in the theory of the evolution of stellar nebulae into dumb-bell form, with their ultimate consolidation into "double" stars.

extremes of propinquity, or concentration of matter, when measured between planets.

It is necessary to point out, however, that a similar range in the mean energetic values of the other factors of energy, such as force, velocity, density, etc., is to be expected as one passes from mechanical to celestial energetics, on the one hand, or to molecular energetics on the other. Yet there is no reason to suppose, from these differences in degree, that the principle of action is at all different. For instance, there has been every reason to suppose that the principles of mechanics were applicable to the explanation of thermal phenomena. And yet, if so, how are the enormous forces which are needed to account for thermal energies to be explained? Thus, one pound of carbon, in the form of coal, by virtue of its separation from $2\frac{2}{3}$ pounds of oxygen in the atmosphere, embodies in the total mass of $3\frac{2}{3}$ pounds some 14,600 B.t.u. or 11,360,000 foot-pounds of potential energy. All of this is released kinetically upon letting the carbon and oxygen fall together, or "burn." Equating this quantity to the equation for potential energy, in which the quantity-factor M_1M_2 would equal $\frac{1}{32.16} \times \frac{2.666}{32.16} = 0.00258$, it results that S_0, the closest distance of approach between carbon and oxygen, must be only about 10^{-16} inches. But this is much closer than the mean diameter of molecules, as estimated from other sources, has been accepted as being. According to Professor J. J. Thomson, the diameter of the molecule is about 4×10^{-9} inches, while that of the electron is about 4×10^{-14} inches.*

From this the conclusion is driven inevitably to one of two things. First, the carbon and oxygen must each be subdivided into particles much smaller than a single molecule, which so penetrate the other's molecular swarms as to attain a mean propinquity far greater than that permitted by molecular diameters. Secondly, the density of mass in the molecular particles is far greater than that of matter as we are familiar with it in the solar system. Probably both statements are true.

This is the equivalent of saying that what has hitherto been considered and measured in diameter as a molecule is not to be considered as a solid homogeneous sphere, but as a multiplex

Engineering (London), March 19, 1909, page 391.

mass-system, containing much space as well as material particles. It was Professor Rowland, we believe, who said, many years ago, that "a grand piano, in comparison with a molecule, is simplicity itself." As to what may be the proportions between space and matter in the molecule, nothing may be said with confidence. Pursuing, however, the plan hitherto followed, of deducing all our molecular ideas from celestial mechanics, the molecule should follow the proportions of our solar system.

But in speaking of densities, in connection with solar systems and molecules, it is too commonly assumed that the densities of individual planets, such as our earth, constitute our sole and proper guide to the densities of molecular matter. But if molecules are to be likened to solar systems, a collection of molecules, such as ordinary matter, must be likened to a collection of solar systems. The density of the particles composing the molecule must bear the same proportion to the density of the entire molecule, or to a collection of molecules, as the density of planetary mass bears to the mean density of a solar system or a galaxy.

What, then, is the mean density of our solar system? Assuming that the orbit of the most distant planet, Neptune, constitutes the outer limit of the solar system—although many comets controlled by our sun far exceed these limits—and remembering that a molecule is a system occupying space of three dimensions, whereas the orbits of the planets occupy virtually only space of two dimensions—the mean density of the space occupied by the solar system is easily computed as being less than that of our solid earth by about the ratio 10^{12}. This means that the mean density of such a system, when perceived from far enough without so that it appears as a unit, is about that of one three-hundred-millionth of an atmosphere, or far rarer than the matter within an electric-light bulb, or even a Crookes tube. If these same proportions are to apply to a molecule of carbon dioxid, for instance, then the mean density of the smallest particles which engage in heat-motion, and which will retain the characteristics of carbon and oxygen, respectively, must be some million millions of times greater than that of our solid earth.

There is no intention here to place any weight upon the exact arithmetical aspect of these proportions. All that is intended is that, if our ideas of heat-energy are to be explained in terms of mechanics, and if our mechanics are to be drawn from the only

known source of true mechanics—celestial mechanics—then there appears to be no difficulty in explaining the heat-energy stored chemically in the dissociation of elements, as due to simple gravitation between mass-particles. It is just as probable that the density of such related particles is many millions of times greater than the densities of matter familiar to human discernment as it is that molecular dimensions and distances should be many millions of times smaller. It is the wide range in densities thus opened before us, as we deal with more and more finely subdivided mass, which makes it possible to explain even the energy of radium, which embodies a million times the energy of an equal mass of oxygen and hydrogen, in terms of mechanical energetics. Similarly, the explanation of the heat-energy stored in the physical disgregation of matter in vaporization or fusion is still easier, for the amount of energy stored per unit of aggregate mass is much less. If it should happen that not all of the vast range of dimensions which the above figures show to be open to the imagination is necessary for the explanation, so much the better. If it should happen, too, that the dimensions, densities, etc., which are requisite for the mechanical explanation of heat do not immediately fit the estimates which have been drawn from other considerations, the discrepancy is for the physicists to explain. The doctrines of the Newtonian mechanics, and of heat as a mode of motion and separation within mass, are both of them now too broadly founded in scientific experience for them to bend easily to meet other empiricisms which may be discrepant therewith.*

A second intention of these statements of enormous ratios is to impress upon the mind the fact that the solid forms of matter with which the mechanics of engineering is chiefly concerned

*The writer wishes to repeat and emphasize here the caution which appears in the preface, that these statements are not to be regarded by the student as coming from one who has investigated molecular activities at length in the physical laboratory. They are founded merely upon the premises which have been frequently repeated throughout the work, as appertaining to the whole—the Newtonian mechanics and the doctrine that heat and work are one. But, in oral discussion of these matters with those who are entitled to speak as physicists, he has frequently met the objection stated, that it was impossible to explain the gigantic forces and energies of molecular mass mechanically. But in every case, upon investigation, it has developed that this position has been based upon some quite unwarranted and limiting assumption—such as this that molecular mass must be of densities similar to mundane density—which is in reality gratuitous and unnatural. It is to show that such assumptions are not only needless, but groundless, that this argument has been inserted.

constitute, when viewed in proportion to the linear distances and volumes also employed, a most extreme condition of concentration of matter. If these engineering bodies were to be located properly on the field of Fig. 8, or Fig. 12, for instance, their place would be found at the extreme prolongation of the curve CB beyond B. That is to say, iron, steel and granite, as used in machine construction, are about as far removed from their natural mean energetic conditions as ice, when at a temperature of —400° Fahr., is removed from that condition of boiling water or superheated steam wherein it displays its greatest thermodynamic adaptability for handling large quantities of energy, both in thermogy and labority—the condition in which it explodes our boilers, drives our engines and ameliorates the sudden changes of climate and season.

It now appears that a finally exact definition of heat is a very difficult thing. In the Seventh Paper it was defined as the spacial and kinetic relativities between the particles of a body, *excluding* the subpermanent or colliding particles. Now it seems that the definition of even so simple a term as collision is difficult. Collision is an approach so close as to upset the permanence of equilibrium of molecular existence. A few years ago we should have said that this settled it: We knew what permanence meant. Now questions are raised as to the permanence of existence of chemical matter, by the degradation of radioactive matter, which are hard to answer; and heat can scarcely be a more permanent form of energy than chemical energy.

Moreover, little light has yet been shed upon the question whether the superpermanent, or hyperbolic, energies of the molecule should be included as heat. To the writer, they plainly should.

It will be found of great assistance to the understanding of heat-action, in the next paper, even if we do not know just what heat is, to have done these things, viz: (1) To have disposed of the idea of the "perfectly elastic yet solid" molecule; (2) to have reduced the definition of contact and collision to their proper places; (3) to have similarly disenthroned the usurper called "perfect gas"; and (4) to have established heat-action similarly with mechanical action upon the basis of the mean energetic condition, depending upon action at a distance, rather than upon that natural unreality, contact.

CHAPTER XI.

The Two Basic Thermal Processes.

HEAT-TRANSFER AND WORK-PERFORMANCE.

Chapter IX announced the need, in thermodynamic discussion, of mechanical concepts of pressure, volume, temperature and entropy. The first two of these obscure properties of matter have now been discussed, in their mechanical aspect. Before proceeding to a similar discussion of temperature and entropy, however, it will be necessary to develop the mechanical concepts of the two basic thermodynamic *processes,* heat-transfer and work-performance, as contrasted with static thermal *attributes,* such as pressure, volume, temperature and entropy.

The thermal diagram, Fig. 8, which was presented in Chapter VIII (page 100), developed before the eye only two out of the several simple thermal processes which are familiar to boiler and engine-room. These two were, first, the *isomorphic* and *metathermal** heating and cooling of bodies. This process was explained as if it were performed only by heat developed by impact and friction; but heat supplied by conduction or radiation would have been found to produce identical results. The second process was the *metamorphic* and *isothermal* one of addition or abstraction of heat when the body was neither heated nor cooled, but changed its physical state instead, as in fusion or vaporization. This too was explained with heat furnished by impact or friction, although heat produced by conduction or radiation would have produced the same results. In the latter case, however, the original form of the energy would have been as obscure to the understanding as the final one (whereas we feel that we understand the mechanical nature of impact or friction) and so would have been of little aid to the understanding.

The familiar thermal phenomena of the power-house, however, include at least two other processes which need explanation.

*Meaning "temperature-changing," contrasted with *isothermal.* "Meta" signifies change, as "iso" signifies constancy.

These are, first, adiabatic work-performance by heat, and, secondly, the isenergic degradation of heat by wire-drawing. It will be assumed that the reader is sufficiently familiar with both of these processes to avoid the necessity of their general description. However, in order to make prominent the features which are sufficiently characteristic to aid in understanding their internal mechanical operation, they will be briefly defined.

The Adiabatic Process. Adiabatic action occurs only when a body undergoes simultaneous change in volume and temperature, while expanding or being compressed, with the exclusion of heat-interchange by conduction or radiation or friction or impact with other bodies. The only permissible exchange of energy with the outside world is in the form of *work;* and this exchange is unavoidable, if the volume is to alter, for pressure exists everywhere.

The energy thus exchanged is supplied from the body's fund of heat; and not only from its fund of heat, but from its fund of *temperature*-heat. For no other sort of heat is available for work-performance. Not only does adiabatic work-performance always result in temperature-change, but *it is the only known exact measure of temperature-change.* This fact is the basis of Carnot's foundation of thermodynamics, and of Lord Kelvin's perfect scale of temperature, in which equal "degrees" are defined in terms of equal amounts of work performed, not in terms of equal amounts of heat transferred; nor, as in our ordinary thermometry, in terms of equal increments in volume of expansion.

In adiabatic action the alterations in volume and temperature occur in inverse directions. The temperature decreases as the volume increases, and vice versa. The process is represented in the thermal diagram, Fig. 8, by a straight vertical line.

The Wire-drawing Process. When a fluid, whether gaseous or liquid, flows through a pipe or orifice, the flow is resisted by friction. The thermodynamic effect of this takes either one of two forms.

1. If the pressure upon the substance be greater than its vapor-tension, for all the temperatures in question, the temperature of the fluid will rise and its pressure fall as it proceeds. This process is an isomorphic one of heating by friction and impact, as already fully described, except that it occurs under

falling pressure. Such is the action, for instance, in all water-pipes of ordinary temperature.

2. If the pressure at any point should become less than the vapor-tension corresponding to the local temperature (and it is obvious that Process No. 1 tends to result in this state of affairs, if carried far enough), then the wire-drawing process ceases to be isomorphic and becomes metamorphic. The substance begins to vaporize. Such, for instance, is the action in the ordinary boiler-blow-off, particularly if the blow-off pipe be imagined as so long that virtually all of the energy of the escaping fluids is spent in overcoming friction. The same action occurs in the expansion-valve of the ammonia refrigerating-machine.

Or, if the substance happens to be already vaporized when it starts upon its impeded flow, as is the case with steam flowing through a steam-pipe, then the result of the isenergic action will be to further rarefy it. This action is familiar in the ordinary throttling and superheating calorimeter.

Between these two numerated forms of isenergetic wire-drawing there exists a most instructing similarity and contrast. The similarity lies in the fact that *both processes increase the quantity-factor of the heat*, or its *entropy*. The contrast lies in the fact that whereas the first process *in*creases the temperature, the second *de*creases it.

The conclusions to be drawn are obvious, and are two-fold.

1. Friction and impact are intimately and functionally linked with entropy-change.

2. Frictional or impactive increase of entropy occurs independently of, and has no determinative effect upon, temperature. The question as to whether the temperature is to rise or fall or remain constant, as the result of friction or impact, depends solely upon the external pressure. For, as has just been pointed out, the increase of entropy by impact or friction under rising, falling or stably constant temperature are processes equally familiar in nature.

In order to be sure of understanding this position, it were well to examine further the nature of wire-drawing.

Taking, for convenience, the second, or expansive, form of wire-drawing, it soon reveals itself as a duplex process. It consists, in reality, of two distinct processes merged into an apparent, but not real, unity. First comes the acceleration of the mass

of liquid or gas into linear motion. This absorbs work; and as the only source of energy for this purpose is the body's heat, the temperature falls to supply it. This part of the wire-drawing process is purely adiabatic.

But the linear motion of the particles is no more than generated than it engenders friction; and the word friction is merely our short name, as has been seen, for the destruction of linear motion in the increase of the quantity-factor of heat. In the present case, since the rate of flow is in equilibrium with the friction opposing it, the rate of energy-transformation, all around, is controlled by the degree of friction present. The rough, hard walls turn the work of flow into heat as fast as they can, and the consequent deficit in mechanical energy is made good continuously by the further adiabatic expansion of the body.

Now it happens that wire-drawing is our only instance of simultaneous adiabatic expansion and friction. The steam-turbine seems at first to offer another and more illustrative case. But upon second thought it appears that its internal action is the same as pure wire-drawing, except that it is complicated by the simultaneous presence of still a third process, viz: adiabatic work-performance upon outside systems. In steam-engine cylinders we have an obverse illustration, viz: the addition or abstraction of heat while adiabatic work-performance is going on. But to bring this into line with pure wire-drawing we must introduce the inference, which is everywhere forced upon us, that friction and thermal conduction are identical processes, in their ultimate nature; except that whereas conduction will work both ways—in and out—friction will work only one.

Nevertheless, these facts serve only to broaden, rather than to narrow, the conclusions stated above, viz:

1. That friction affects only entropy, being indeterminative of temperature-change;

2. That the direction of temperature-change depends solely upon whether there be a surplus of external over internal pressure, radially active, or not; and

3. That in the apparently single process of wire-drawing there really exists a merger of both the above independent processes, viz: a variation of the entropy by friction, and a variation of the temperature by pressure-action or work-performance.

If we turn now from the development of heat by friction or

impact to its twin brother, the development of heat by thermal conduction, there appears a most striking similarity to the wire-drawing situation. That is to say, not only does thermal conduction identify itself, by ways not necessarily repeated here, with friction and impact as an inevitable controller of entropy-change, but *the temperature-change which accompanies it is not determined by the quantity of heat conducted, but solely by the relation between external and internal pressures.* If the substance be water, for instance, and if the pressure upon it be greater than its vapor-tension, then the addition of heat by thermal conduction, or by friction either, *in*creases the temperature—or rather, it results in an increase in temperature. But if the external pressure and the vapor-tension be balanced, as is the case in the ordinary steam-boiler, then the addition of heat by thermal conduction, or by friction either (though in this case we lack a familiar instance), results in *no* rise in temperature. The water merely vaporizes isothermally.

If, again, the external pressure happens to be less than the vapor-tension—as is the case in a steam-engine cylinder when the resistance of the piston is less than the expansive force of the steam—then the addition of heat by thermal conduction, such as occurs if the engine-cylinder be steam-jacketed, occurs in connection with a *fall* of temperature. Or the wire-drawing of steam, already described, is an even better illustration.

We have finally come to the point, therefore, where the absurdity of calling temperature-change the effect of either friction or thermal conduction has become obvious. It is entropy-change alone which is this. Temperature-change must be the functional result of something quite different. In cylinder-expansion we need no assurance that it is the deficit of *pressure*, with the consequent retreat of the confining walls, which creates the drop in temperature. The simultaneous addition of heat has nothing to do with the case, except in the alteration of the entropy present. But in the addition of heat to the water in a boiler, thermal conduction has no more to do with the temperature-*rise* than it has, in the steam-jacketed cylinder, with the the temperature-*fall*. It is pressure alone which controls both.

It is only because the great majority of all the things to which we have been accustomed to add heat have had vapor-tensions below that of the surrounding atmosphere, or other

external pressure, that we have come to associate the addition of energy by thermal conduction, or by friction and impact, with rise of temperature. This popular association of these two phenomena is not only loose and inaccurate, but it is fundamentally erroneous and misleading. It begets in the student's mind an idea of interdependence between temperature-change and thermal conduction which is in direct antithesis to the principle laid down by Lord Kelvin, viz: that the only correct measure of temperature-change, the only correct thermometric degree, is *work-performance.* It draws the very foundation from beneath what should be the first and fundamental idea taught the student in the thermal laboratory, viz: that thermal conduction has everything to do with alterations in entropy, and nothing whatever to do with alterations in temperature.

The deliberate association of thermometry and thermal conduction in the physical laboratory should cease. The first efforts of the instructor in thermodynamics, after a clear concept of the general nature of energy is once implanted, should be directed toward a breaking up of this false association of ideas, already too deeply imbedded by the ordinary experiences of life before reaching the study of the natural sciences.

ONLY TWO FUNDAMENTAL THERMODYNAMIC PROCESSES.

The conclusion is inevitable, therefore, that *there exist but two basic, elementary, thermodynamic processes.* One of these is the generation of thermal quantity-factor, or entropy, by impact, friction, thermal conduction or any other process which proves upon examination to be an equivalent: a process which always occurs at constant temperature, or independently of and ineffectively upon the temperature.

The other process is the adiabatic variation of temperature by work-performance: a process which always occurs at constant entropy, or independently of and ineffectively upon entropy-change, but which cannot occur independently of temperature-change.

In order that the fundamental importance of these two processes may be clearly brought out, they may be given distinctive names and completely defined, as follows:

1. THERMOGY: the isothermal variation of entropy, occurring always when matter is subjected to impact, friction, radia-

tion, thermal or electrical conduction, or perhaps other processes as yet unidentified. In the mathematical discussion of thermogy the temperature T must always appear as an integer, and the entropy-change dN as a differential.

2. LABORITY: the isentropic, or adiabatic, variation of temperature, occurring always when matter develops work from heat, and sometimes when matter develops heat from work—that is, when it absorbs work radially with perfect elasticity, or "reversibly." In the mathematical discussion of labority the temperature-factor dT must always appear as a differential, and the entropy-factor N as an integer.

The first of these processes must always be represented in the thermal diagram, Fig. 8, by a straight *horizontal* line, properly of infinitesimal length, but integrable, under proper conditions, into a finite extent. The second process must be represented by a straight *vertical* line, under similar conditions.

All other thermodynamic processes whatever, of which several are commonly depicted upon the thermal diagram by various oblique and curved lines, must be considered as made up, in natural fact as well as geometrically, of these two basic components, occurring simultaneously and combining to produce an apparently single, though really double, result.

Thus, in the isomorphic heating of water under pressure, which is represented by the oblique curve ADW, Fig. 8, the energy is supplied in such form (either by friction, impact, conduction or radiation) that the quantity-factor is increased by the value dN, say. The quantity of energy thus supplied, dQ, is equal to T dN and is represented by a vertical element of area between ADW and OX. The result of this supply of energy is to vaporize momentarily a minute quantity of the water.

If the external pressure upon the water be exactly balanced with the vapor-tension, as along DE, this minute portion of vapor remains vapor; and further increments of vapor, from further increments of heat, also remain vapor, cumulatively. But if the external pressure be in excess, as is necessary for the prosecution of the isomorphic AD toward D′, the tiny bits of vapor are recompressed, adiabatically, into water, as fast as they are produced, leading to the adiabatic rise in temperature dT. The result of this simultaneous increase of quantity-factor and temperature is portrayed by the curve DW.

The other isomorphic curves might be similarly explained, except that in the case of ice the reaction cannot occur between solid and liquid forms, because pressure turns ice to water, not water to ice. The only way out of the difficulty is to assume that there must be present in the ice, at all temperatures, minute quantities of both water and water-vapor, between which the reaction occurs; and this fact has been developed independently by direct observation. In the case of refractory solids, which refuse to fuse under all ordinary applications of heat, it is to be remarked that the resistance to fusion is plainly involved in the crystalline form of the solids—a question too complex and bearing too lightly upon power-engineering for discussion here.

Throughout the previous discussion the quantity-factor of heat has appeared as a matter of fundamental import. Obviously, it also needs a short and suitable name. Hitherto it has been referred to often merely as "the quantity-factor," in order that the discussion might not be vitiated by doubts as to the propriety of a name, or by preconceived notions as to the nature of entropy, the only name suitable for it. But the analysis has now reached a stage where these considerations need no longer defer the identification of the quantity-factor of Fig. 8 with the term *entropy* which was invented by Professor Clausius forty years ago. Fig. 8 itself may be similarly identified with the entropy-temperature diagram of Gibbs and Macfarlane Gray.

It should be explained immediately that this identification is not complete nor exact nor final; but this is so only because doubt exists as to the natural facts upon which the definition of entropy rests. Professor Clausius defined his understanding of entropy as being a function "equal to or greater than, but never less than, the integration of $\frac{dQ}{T}$" in any thermodynamic cycle. The equality was to apply when the cycle is free from impact and friction, or was "reversible." The increase in entropy would occur when these were present—as they always are.

But Clausius's definition of entropy, while deserving the very great credit of being the first to appear, is nevertheless incomplete. It is incomplete in regarding itself as complete and final. It is incomplete in three distinct ways.

First, while Clausius's definition of entropy is exact enough in terms od dQ, *he gives no definition of dQ.* None of the text-

books which have followed Clausius attempt to define it. We are still to-day, although possessing a vastly greater accumulation of data than in Clausius's time, unable to define dQ. Clausius defined dQ in terms of impact, friction, radiation and conduction. We know now that that list is incomplete; but what to add to it (beyond some types of chemical, electrical and magnetic action), and how we are to know when we have finished the list, we are yet unable to say.

Secondly, Clausius recognized no other form of entropy than thermal entropy. In Clausius's day, although the Conservation of Energy had been foreshadowed by Newton (1680) and defined and pictured pretty completely in the writings of Mohr (1837, Mayer (1843), Grove (1846) and Helmholz (1847), two decades before Clausius, yet the idea that heat is a mode of motion was still obscure in the scientific world. It was far beyond the natural, for that day, for Clausius to see that when he had identified entropy-production with impact and friction, which are purely mechanical, he had proven that there must be a mechanical form of entropy, even if there were none of heat or temperature. Thus, long before the brilliant work which began with Count Rumford and ended with Kelvin, Joule and Maxwell had settled beyond peradventure the physical reality and the mechanical nature of both heat and temperature, the real (albeit undeveloped) foundation for the identification of entropy also, as a physical reality and a feature of mechanical energy, had been laid, in the original discovery of the function itself.

To-day the doubt lies not as to whether entropy be a physical reality, such as temperature, heat or work, rather than a mere mathematical function. It does not even lie in the direction of whether there may be other sorts of entropy than thermal entropy. Our difficulty to-day is to know where the concept of entropy, as a physical reality, is to end. We know that as far as energy-transformations extend, so far do all of the forms of energy exhibit the dual attributes of intensity and extensity, or quantity-factor, respectively. If the energy itself is identical throughout all these changes, so must be the two component factors. There must be as many forms of temperature and entropy as there are forms of energy interchangeable with heat. But for their exact definition we must await further knowledge.

Thirdly, Clausius saw no chance for the entropy of any

system ever to decrease. Knowing no other form of entropy than thermal entropy, this was most natural; for thermal entropy, so long as it stays thermal, does tend constantly to increase—upon the earth's surface, at any rate. But more, so far as scientific thought may surmise, everywhere in nature exists some solidity, or at least viscosity; and wherever solidity or viscosity exists, mechanical motion is being constantly converted into heat and heat is degrading itself in temperture, with increase of entropy.

Starting from these facts Professor Zeuner announced his theorem that the entropy of the universe tends always to a maximum. Proceeding further from this same base, Lord Kelvin gave countenance to the cosmic doctrine of the general degradation of all forms of energy into unavailability; and the school of what is now orthodox thermodynamics was settled beyond discussion.* Whether there be such a thing or not as a universal degradation of energy will be discussed in a later chapter. The writer may announce here that he does not believe that there is. He believes that the availability of the energy of the universe remains constant, although in regions as great as a solar system it may locally and temporarily fall into deficit or accumulate into a surplus.

But it is not upon any principle so broad as this that the concept of entropy must rest. It rests upon the familiar, every-day transformations of energy, and upon the fact that therein

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*The following extract is taken from a lecture by Sir Oliver Lodge upon Lord Kelvin, which was delivered after these pages had been written:

"I fancy that he himself, and certainly some of his disciples, have been at times inclined to attribute to the law of degradation more ultimate and cosmic importance than properly belongs to it. Its significance is limited to the validity of the terms 'heat' and 'temperature'; and if for any reason these cease to have a practical meaning, then the dissipation of energy ceases to be inevitable. * * The different availabilities of various kinds must be essentially a human and temporary conception, useful and convenient for practical purposes, but not ultimate nor cosmic. * * The dissipation of energy has no meaning when 'heat' and 'temperature' are obsolete terms; that is to say, when what we now consider to be unorganized and intractable molecular motions can be dealt with in an individual and organized way."

Professor Zeuner, too, is to be credited with the first perception of the physical reality of entropy. His term for it, "heat-weight," is a most graphic expression of one of the prime characteristics of entropy, namely, its "heft" or forcefulness in work-performance. But entropy, we shall see, is not the force of thermal gravitation, but the degree of subdivision of matter which develops that force, and which, taken with propinquity, determines its magnitude.

appears everywhere this duality of intensity-factor and quantity-factor of energy, running through all the natural sciences. Its importance is basic, not only in the cases of heat and work, which alone can be discussed here, but in chemical, electrical, magnetic and radiant energies as well. Although it is only in mechanical and thermal energies that the quantity-factor has been accurately defined—as $\Sigma\ (M_1M_2)$ or the extent of mass-pairing present in the first, and as entropy in the second—yet in all the other sciences its existence and importance are none the less indubitable because vague. In all these fields it demands immediate definition. Not only must the engineer, the electrician and the chemist contribute to the physicist their data, before this general energy-factor which in heat finds the name entropy can be properly defined, but the biologist, the economist and the sociologist must follow suit. In all of these sciences there has been too much time devoted to study and measurement of the visible space-and-motion factors, and far too little attention given to understanding what it is which can experience space or motion. There need be no hesitancy in stating that when our students are properly taught the nature of entropy, in its broadest sense, they will find the concept of equal use, in after life, in their science, their engineering, their business and their politics.*

For all these reasons the writer will use the term entropy in a considerably wider, and a somewhat different, sense from that given it originally by Professor Clausius (if a bare and rigid mathematical definition can be called a "sense"). He will use it as synonymous with the quantity-factor of heat, and he will show in the next paper that it is identical with the quantity-factor in mechanical energy, $\Sigma\ (M_1M_2)$. He will also suggest briefly the idea that this same quantity-factor runs through all the other forms of energetic action to which the laws of the transformation and conservation of energy apply, in the inanimate, vegetable, animal, human and social activities of the universe. It will appear that no student may lay claim to a grasp of the fundamental principles which guide and control any of these activities, without first acquiring a thorough comprehension of the mass-factor in mechanical energy—that it is the form of

*The reader who desires a further development of this idea will find its discussion in the *Harvard Engineering Journal* for 1908, January and November issues. The broader significance of the quantity-factor of energy receives further discussion in the last chapter of this book.

association, or grouping, or relationship, between things, ***and not the things themselves,*** which gives rise to energy, and to the characteristics of its results.

The next thing is to understand this corresponding thing in heat which we call entropy. The final fruit is to comprehend that all forms of energetic activity, including social energetics, are based upon the same factors and follow the same laws as those which support and guide the silent stars, the incandescent flame and the awful thunderbolt.

CHAPTER XII.

Mechanical Concepts of Thermal Phenomena.

B. TEMPERATURE AND ENTROPY.

The definitions given in the preceding paper for the two basic thermodynamic processes, thermogy and labority, now pave the way for clear mechanical concepts of temperature and entropy. Before attempting to formulate any such concepts, however, it were well to get well in mind the peculiarities which are characteristic of the two quantities respectively.

Temperature. The prime characteristic of temperature, and the only true measure of temperature-change, is *work-performance.* Work can be performed at the expense of heat *only* when the temperature falls. Work can be absorbed elastically, with obvious conservation of energy, or "reversibly" as it is called, only when the temperature rises.

This idea of temperature we owe originally to Carnot (1825), who stated the law that the work performed—if the process were perfectly elastic, or "radial," or "reversible"—was proportional to the temperature-fall; that is, the law is conditioned upon pure labority, devoid of thermogy—the straight, vertical, adiabatic process. Half a century later this idea was amplified by Lord Kelvin into the principle that the only accurate measure of temperature-variation, or the only true thermometric scale, was to be based upon equal degrees, not of heat-supply, nor of volume of expansion, but of *work-performance.*

The second characteristic of temperature is the universal tendency of differences of temperature to set up energy-transfer by thermal conduction and radiation. This is the explanation of the familiar sensations of hot or cold—the gain or loss of energy by the skin and nerves when brought into contact with other bodies, or when exposed to radiation. These ideas of temperature long antedated the discovery of the work-scale of temperature; yet they are by no means such accurate guides in the understanding of the nature of heat. They must be con-

sidered quite secondary to the prime characteristic of temperature-heat: work-performance.

These sensations are deceptive because they concern merely the *rate* of transfer of energy, a thing which is subject to other influences than mere temperature-difference. Thus, exposure to the wind, or wetting the skin, will heighten the sensation of cold. The latter of these two processes may actually lower the temperature and be perceived by the thermometer; but the former makes no difference to the thermometer.

A still more accidental characteristic of temperature is its effect upon the volume of a substance. All gases and vapors increase in volume with rising temperature. Some liquids and solids do the same, though at a lower rate; but many reverse the rule and increase in volume with falling temperature. Yet this characteristic of temperature is our main reliance in ordinary thermometry. The relative expansion of mercury and glass is the most common means for exhibiting temperature-changes. For more accurate purposes hydrogen is substituted for the mercury.

A similar characteristic of temperature is the proportionality to it of the expansive pressure of gases and vapors. This proportionality, like that of volume, is never strictly true, even for gases, and for saturated vapors it is quite approximate. In solids and cold liquids the vapor-tension becomes an insignificant affair. While neither pressure nor volume is ever exactly proportional to the temperature, the very fact that the departure from proportionality is least and almost zero in the gases, and is greatest in the solids, is itself a guide to the understanding of temperature—as will be developed later.

Entropy. The prime characteristic of entropy is its *indifference to reversible or elastic work-performance,* by or upon the body, with accompanying temperature-change. Its secondary characteristic is its *sensitiveness to and inseparability from impact, friction, conduction and radiation.* Its third characteristic is its general proportionality with volume. While the relations between entropy and volume are even less regular than those between pressure or volume and temperature, yet they are obvious. They stand out most plainly in the process of vaporization, wherein the increase in both volume and entropy is large, and is exactly proportional.

The last characteristic of entropy to be mentioned, but not the least in importance, is its general proportionality to elasticity. This, while inexact, like volume, is yet seldom reversed. An increase in entropy almost always means an increase in elasticity. It is this fact which maintains thermodynamic happenings in an equilibrium which is almost always stable. Impact and friction are features of inelasticity. Impact and friction always develop entropy. Entropy is always accompanied by elasticity. Elasticity reduces impact and friction. Thus, the phenomena which are due to a deficit of elasticity result in the cancellation of that deficit and a retardation of the phenomena.

Labority and Thermogy. The facts needed to complete the data for the mechanical definition of temperature and entropy are those of the two basic thermodynamic processes already defined in the preceding paper, viz: labority and thermogy. Labority, or work-performance, is identified with temperature-change, but does not affect entropy. Thermogy is identified with entropy-change, but does not itself affect temperature.

Radial and Tangential Action. It is next to be noted that work-performance by heat is always accomplished by an elastic movement of the surface of the hot body normally to itself; that is to say, *radially* in reference to the molecules composing the surface-layer. And if the expansion of the body is supposed to be effected by a simultaneous expansion of all of its molecules, this too would obviously be a radial movement on the part of each one.

Thermogy, on the other hand, consists of an inelastic movement of the surface of the hot body in its own plane; that is to say, *tangentially* in reference to the molecules of the surface-layer. In the case of rubbing friction, this fact is obvious. In the case of direct impact it is less obvious. But even here it is only necessary to note that impact consists, by its very definition, of the energy which is *not* returned radially or elastically, to see that impactive energy must also be tangential in its mode of transfer.

As to conduction and radiation, all that can be said is that nothing positive is known as to the form of their action, but that their results are identical with those of impact and friction. Until positive evidence to the contrary arises, therefore, it is

natural to assume that impact, friction, conduction, radiation, and even electrical resistance, are all identical, as to their form of molecular transfer; that is to say, that they are all tangential, and not radial, activities.

It is now to be recalled that in the study of the elementary mechanical mass-pair there was found a marked contrast between the radial and tangential funds of purely mechanical energy. The radial fund was easily defined with mathematical accuracy. It was the *perceptible* fund of energy. It constituted the medium of communication with outside mass-systems. Its intensity was what determined whether the energy should undergo transformation or not: kinetic intensity above the critical point producing dissociation; spacial intensity (of concentration, or propinquity, or $\frac{1}{S_0}$) above the critical point producing collision.

The tangential energy-fund of the pair, on the other hand, was found to be incapable of exact definition, although alterations in it could be measured exactly. It was the *imperceptible* or latent fund of energy. It was self-contained within the mass-pair. It carried on no direct communication with outside systems. Nevertheless, it was capable of receiving or sending energetic messages to the outside world, through the medium of the radial energy, to an indefinite extent.

Mechanical Concepts of Temperature, Entropy and Heat. The data and mechanism for the comprehension of thermodynamic happenings, as activities of mass, motion and space, are now before the eye, fairly complete.

Heat consists in both motion and space between the molecules of the hot body. Each molecule is itself a complex system of particles, possessing motion and space relatively to each other; and these internal relationships are also heat, in part at least. We are not attempting to define the molecule itself, nor the atoms which form parts of a molecule, nor the ions or electrons which may form parts of the atoms. That is for the chemists and physicists to do. All that is being said here is that, *if* heat is to be regarded as a mode of mechanical motion at all, it must be regarded as a very complex system of motions and spacial separations, some of them between the molecules, and some of them within the molecules. In the gases the motion is chiefly *between* the molecules, each molecule moving bodily along hyper-

bolic paths which are almost straight lines; but some of the heat-energy still remains within each molecule. In the solids the motion is chiefly *within* each molecule, but there is still some motion between each two molecules, of tiny projectile particles if not of the whole molecule.

These relative motions are of two sorts, or components, viz: radial and tangential. Each orbit contains some of each. The elliptical orbits have more tangential component than radial, the hyperbolic orbits more radial than tangential.

Each molecule contains a nucleus of particles moving in elliptic orbit, and a swarm of satellites moving in hyperbolic orbit. In the solids the nucleus is the major feature, as to mass, and the satellites are the minor; in the gases the satellites are the major and the nucleus the minor. In fact, vaporization may consist in the nucleus becoming satellitic. In the so-called "perfect," or absolute, gas the entire mass would be satellitic. In the absolute solid the entire mass would be nuclear. Neither condition is ever attained in nature.

Volume. Each tiny mass-pair embodies a certain degree of spacial or radial separation, which vibrates constantly above and below its "mean energetic distance." In the mean energetic condition the gravitational or centripetal attraction is balanced against the centrifugal force of tangential motion. The mean energetic distance of separation at which this equilibrium is found determines the *volume* of the hot body.

In the solids the mean energetic distances are very small and the *tangential* velocities very high. In the gases the distances are very large (comparatively speaking) and the tangential velocities low. For at larger radii smaller tangential velocities are sufficient to maintain equilibrium.

The *radial* velocities, on the other hand, vary in the reverse order. In the solids they are low and in the gases they are high.

Temperature and Pressure: In this intricate swarm of particles of all sizes, moving in orbits of all dimensions, forms and velocities, the net or integrated effect of all the *radial* activities is both temperature and pressure. The integrated intensity of energy is temperature. The integrated momentum is pressure.

Matter, even in its most quiescent states, finds itself always in contact with other matter, in solid, liquid or gaseous condition. At the point of contact both temperature and pressure are

exerted. If a temperature-difference exists at the point of contact, energy is transmitted across the gap between the bodies, in the form of heat; but the gap does not move. If a pressure-difference exists at the point of contact, no energy is transmitted across the gap; but the gap itself moves, and energy is imparted in the form of work.

In the first case the energy transmitted comes from the internal, or molecular, energy of the matter immediately adjacent to the point of contact. In the second case the energy transmitted comes either very slightly or not at all from this contiguous matter. In the case of "transient" energy it comes from some more or less distant point, and reaches the point of contact solely as the pressure-energy of a mass which itself moves bodily. In the case of energy developed adiabatically by the expansion of the body in contact itself, it originates all over the body, between each nucleus and its satellites, simultaneously; and is then transmitted "transiently" to the particular satellites engaged in "contact."

When temperature-difference exists at the point of contact, the two bodies *may or may not exchange temperature*. That depends entirely upon the pressures prevailing. The only thing certain is that they *always* exchange entropy. A pure illustration —that is, one where the process is not complicated by the influence of pressure-changes—is the multiple-effect evaporator, in a sugar-works. There heat is transmitted from steam condensing under a higher constant pressure and temperature, on one side of a metallic wall, to water evaporating under a lower constant pressure and temperature on the other side. The wall serves to neutralize the pressure-difference, but performs no thermal office. The two steam-bodies exchange only entropy. Neither undergoes temperature-change.

But they can exchange entropy *only* when a temperature-difference exists. The office of temperature is apparently what might be called catalytic: to effect the transfer of entropy, without being itself affected.

Unfortunately, all of our ideas concerning heat are so instinctively founded upon the sense of hotness to the touch, as the prime criterion of thermal conduction, that this simple phenomenon is hard to understand. We say that a substance is "hot," when we touch it, because we find that the finger has been

heated by it. But if the finger be not heated, any amount of heat might be transmitted to it, from a substance of much higher temperature, yet we should not say that the latter was "hot"—except by inference. Thus, the steam-hot water in a boiler, for instance, would not say, could it speak, when a white-hot oil-flame were set against the boiler-shell, that the flame was hot; for the water would not be scorched thereby, nor even raised in temperature at all; nor affected in any way except to be changed into steam no hotter than the water was before.

Similarly, when one touches a hot flat-iron with wetted finger, the same situation, with its lack of sensation of heat, is approached. We judge that the iron is hot, not because it burns the finger, but because it makes a "siss" of steam. We feel no hotness. We merely hear the rapid formation of entropy and volume.

Now this little experiment gives the only true idea of thermal conduction, not as a thing which reveals temperature, but as a thing transmitting entropy—which last we cannot perceive at all, except by its volumetric effects in steam-making. The fact is most difficult to grasp, because all our lives we have grown accustomed to telling whether things were hot or cold by trying if the finger were heated or cooled in touching them. Nor have our physical laboratories, where Lord Kelvin's ideas are well known, done all which it seems they might to teach their students better.

Yet the truth of the matter is that the only situation where temperature can really be *felt* is that, for instance, of the steam-engine piston. This piston might, and ought to, be of the same temperature as the steam pressing against it. There would then be no transfer of heat between them. The piston would not be heated. Yet here alone temperature could be felt. The piston, because of the superior temperature of the expansive steam (as compared with that of the same substance on its opposite face), might be moved, could it speak, to say: *"I am propelled;* therefore I am impelled to note that this steam on one side of me is hot!" For, if Carnot and Kelvin knew anything about the matter, it is *work-performance* alone, and not heat-transfer, which is the only true sign of the presence of temperature.

With our ideas thus clarified it may be accepted that the transfer of heat, by the tiny radiating satellitic particles of a hot body, need not be associated at all with the development of tem-

perature in the body affected. Therefore the puzzle as to the mechanical explanation of pressure, temperature and entropy resolves itself into the following simple array of conditions:

1. Temperature has already been identified, in the case of gaseous matter, by the mechanical theory of heat, as the kinetic energy of the active particles.

2. Temperature is manifested solely by *elastic work-performance;* and work-performance always occurs *normally* to the surface of the body, or *radially* as to the molecules.

3. Heat-transfer, by thermal conduction, has proven to be identical in its effects with impact and friction; and impact and friction have already been identified with *in*elasticity, or molecular action *tangential* to the molecule.

4. Heat-transfer takes place without motion of the envelop demarking the bodies between which conduction occurs; and the only way in which satellites flying along conic-section orbits might transfer energy from one swarm to another, *without* motion of the surface bounding the swarm normally to itself, is *tangentially.*

5. Pressure, however, which is accompanied by no transfer of energy across the bounding surfaces, might be exerted *radially* by the flying particles, under the conditions stated.

It is from this basis that emanated the statement, given above, that both temperature and pressure are manifested by the *radial component* only of the motion of the satellitic particles. Temperature is their integrated intensity of energy. Pressure is their integrated momentum. Temperature-heat is the radial component only of the radial energy, which was defined for the elementary mass-system on page 40.

Thermogy. But, further, it can be added that thermal conduction, like impact and friction, is the *tangential* transfer of energy from the satellites of one swarm to those of another. This can be accomplished only by means of the tangential motion remaining at or near the apastron tip of the orbit. Even when a solid steel boiler-shell, for instance, the molecules of which must embody chiefly tangential motion, is heated by a white-hot gas, the molecules of which must embody mostly radial motion, the energy transferred must be regarded as only the tangential component of the gaseous particles. Because the latter possess but slight tangential motion (although plenty of radial energy) there

prevails but a low rate of heat-transfer. The highly eccentric orbits of the gaseous particles, beautifully adapted for elastic work-performance, are very poorly adapted for thermal conduction. Low rates of heat-transfer, per unit of surface and difference of temperature, are broadly characteristic of gaseous substances. Neither the gases, the satellites of which are many, but of too highly eccentric orbit, nor the solids, the satellites in which possess chiefly tangential motion, but are too few in number and small in mass, transmit heat well by contact. It is the liquids, the satellites in which present the most powerful combination of both numbers or mass (entropy) and tangentiality of motion, which are the best thermal conductors.

But this tangential transfer of energy at the apastron tip of the satellitic orbit cannot occur unless there exists a difference of temperature, or radial intensity, between the two swarms. Two bodies manifesting the same temperature, as well as the same pressure, must have equality in both mass and velocity of satellites. But if the temperature of body A be higher than that of B, while their pressures remain equal (that is, if momentums remain equal while kinetic energies are higher in A), then the mass of A's satellites must be smaller and their velocity higher than in B. But the satellites of each swarm approach the other swarm in all directions, chiefly oblique. Radially their directions and momentums are opposed, and neutralize each other. But tangentially many of them may coincide. In that case the particle of more rapid motion, as of A, will overtake tangentially and impart energy to the more slowly moving particle of the colder body B.

Labority. Elastic work-performance by temperature-heat, on the other hand, is just what these white-hot gases are best fitted for; for that is a purely radial action. This fact is seen in the thermodynamic superiority of the gas-engine over the steam-engine. When a body performs work by heat, the radially flying particles find themselves exerting pressure against (that is, revolving about, at the remote end of their orbits) the satellites of molecular nuclei *which are retreating.* They are like tennis-balls thrown at the rear end of a retreating freight-train, or a jet of water impinging against a retreating Pelton-wheel vane. Their direction of motion is reversed; but they return with their radial velocity much reduced.

Their tangential velocity, however, remains unimpaired; for, if the process has been a pure one, there has been no heat-transfer. Their orbits are therefore reduced in eccentricity and increased in mean energetic distance. The molecule has lost its temperature and pressure, but gained in volume. It has lost its ability to impress itself upon outside systems, but it has not lost its entropy, or the mass-factor or quantity-factor of its heat, which gives heft to its energy. It retains to the full its tangential components. Its entropy remains constant. It is no longer valuable for work-performance, but it is still most efficient for heating-purposes.

Since it will shortly be stated that this quantity-factor or entropy is merely the number and mass of the energy-pairs at work, it is to be noted here that there is nothing about this process of work-performance which should lead to any consolidation of mass, or diminution in the number of mass-pairs at work. Work-performance, whether done mechanically or thermodynamically, always tends to occur at *constancy of mass-pairing*. Only interference by collision makes it otherwise.

The obverse of this process, adiabatic compression, is not so obviously explained. It is plain that the approach of each two molecules, enforced by outside power, must decrease their volume. It is easy to see that the increased momentum of the greater number of projectiles then met and reversed, per unit of area, must increase the pressure. But it is not so clear that this must also increase the radial activity, or temperature.

This will be plain, however, when it is remembered that the forces which act upon any satellite at periastron are the result, not only of mass, but of propinquity. When the molecules are compressed each satellite finds itself forced into greater simultaneous propinquity, at periastron, with a greater number of nuclear masses. Its periastron velocity therefore increases; and this velocity, although tangential in direction, has already been pointed out as being largely true radial energy.

There also occurs the direct effect upon the radial velocity of the approach of the external masses about which the satellites are reversed, just as in expansion. Owing to the approach of these masses, like a freight-train or Pelton wheel being backed up against the projectiles which strike it, these are returned with increased radial velocities.

Temperature. Temperature, then, is radial kinetic energy. Work-performance by temperature-drop is in no way different from the performance of work by the velocity-reduction of mechanical mass, as in a turbine or Pelton water-wheel. Temperature-increase by compression is in no way different from the acceleration of any mass by a supply of energy, as in the centrifugal pump or marine propeller. The engineer who can understand these mechanical devices can understand adiabatic thermodynamics.

The doctrine that temperature is kinetic energy is not new, at all. What these papers are intended to emphasize, in connection with it, is that the kinetic energy which constitutes temperature is

1. The radial component only of the molecular energy;
2. It is embodied in mass-particles very much smaller than the entire molecule, as well as in the latter itself;
3. These facts apply to liquids and solids as well as to gases;
4. The "rebound" of the flying particles cannot be imagined as that of perfect elasticity in collision, but must be accepted as consisting of circumrevolution without contact.

Entropy. Turning now to the question of the mechanical nature of entropy, and of its generative process, thermogy, it might be inferred that these were to be found, because of their contrast with temperature and labority, in the *tangential* motion of the particles. They are, indeed, closely connected therewith. Thermogy is the acceleration of the particles in their tangential component. But entropy is the result of this process, not the process itself; and the natural result of an increase in tangential energies is a separation or scattering or subdivision of the total mass into a greater number of mass-pairs capable of embodying temperature.

Entropy is what corresponds to the quantity factor M_1M_2 of the elementary mechanical mass-pair. It is, more accurately, the Σ (MM) of any mechanically energetic system. It is the X of Fig. 7, a function of n the number of portions into which the total mass is subdivided. It is the quantity-factor N of the thermal diagram, Fig. 8. It is the degree of subdivision and dispersion of the nuclei A and B of Fig. 9 into satellitic swarms. It is, in short, the degree of subdivision, specialization and organization of an originally comparatively unified, solid, rigid, inert and inelastic mass, into a system capable of embodying that

space and motion which are to us the sole visible manifestations of energy and elasticity. This quantity-factor is always itself imperceptible; that is to say, directly imperceptible; but it is what gives "heft" and power to that which alone *is* directly perceptible, viz: space and radial motion.

Intramolecular Equilibrium. It is the stability of equilibrium within the molecule which necessitates always a readjustment of either sort of energy, radial or tangential, whenever any alteration is made in the other. Thus, in heating ice or water isomorphically, only thermogy is performed directly. The alterations of entropy occur at constant temperature; the subdivision of the molecules results from speeding them up tangentially, as a potter does his wheel, and not from any radial acceleration or rise in temperature. But the immediate indirect effect of this thermogy, in the face of a superior external pressure upon the molecule, is to upset its internal equilibrium; so that the energy which was imparted tangentially, expanding the molecule as a fly-ball governor expands by acceleration, is squeezed out radially, by the recompression of the molecule adiabatically by the external pressure, into an increase in radial components which we call temperature-rise. The ice or water gets warmer as it is pounded or heated; but this occurs only because (1) it has first been so subdivided, isothermally, by the thermogy, as to have become more elastic and susceptible of compression; and (2) because the surplus pressure requisite for the recompression of this greater elasticity is present. If the surplus pressure be not present the temperature will not rise. It is the adiabatic recompression of the elastic portion of the molecule, by this apparently static external pressure, and not the thermogic addition of energy to its inelastic portion by pounding or heating, which raises its temperature.

On the other hand, when thermogic impact or friction or heat-conduction leads to the melting of the ice or the vaporization of the water, without incidental rise in temperature, it is because the accumulation of internal radial intensity has become sufficient to counterbalance the static external pressure. The equilibrium has become indifferent. The expansion of the molecule by thermogic increase of its tangential velocity now jumps it into a new condition of stable equilibrium, that of saturated steam —like a fly-ball governor breaking some of its resisting links—

and in that condition it remains stably. No reactive compression can occur to raise the temperature, and we therefore say that vaporization occurs isothermally.

In any such a thermogic action the chief absorbent of energy would be the separation, or disgregation, of the particles. It was pointed out in Chapter I that the expression for space-energy was proportional to $\frac{1}{S_0} - \frac{1}{S}$, and that in this expression the value of S_0 had much more to do with the value of the function than did that of S. In other words, when mass is separated the work involved depends much more upon the original degree of concentration, or density, of the mass than upon the final degree of diffusion. This work of separation is called *disgregation-work* —the word disgregation signifying the "scattering of a flock," or the opposite of congregation. This disgregation work is very great for solids and liquids, because of their comparatively great density. They are unusual congregations or concentrations of mass, embodying unusual deficits of energy. For gases it is much less, decreasing to almost zero as the diffusion becomes great. But because mass can never be so widely separated that its mutually attractive force becomes zero, so no gas can ever become so rarefied or "perfect" that its disgregation-work in change of volume ever becomes zero.

In illustration of this mechanical concept of the thermal molecule, the writer has used the simile of a juggler standing before a table carrying a stock of balls. The total stock of balls represents the molecule. The portion which the juggler succeeds in keeping in motion in the air is the satellitic portion. The remainder upon the table is the nucleus. In this simile the nucleus would have no motion at all, whereas in the actual molecule the nucleus possesses plenty of motion, only reduced to an eccentricity below unity. In the simile the "satellites" possess merely elliptic motion, whereas in the molecule they possess hyperbolic motion. But still the simile may help the understanding; and in it the juggler's energy of action must be supposed to be the energy inherent in the balls themselves.

When the juggler exhibits little energy only a small portion of the balls will be kept in the air at once. The bulk of the stock of mass on hand remains on the table; that is, in the central nucleus of the molecule. As the juggler gains energy, however,

the flying balls will remain longer in the air, and the idle stock on hand must be drawn upon in order to supply fresh recruits. In this simile, the total energy of the flying balls (including their potential energy) is the *heat* of the molecule; their vertical kinetic energy per unit of mass is its *temperature;* the distance they fly is its *volume;* the force with which they might impinge upon a ceiling overhead is its *pressure;* and the degree to which the total stock of balls is subdivided into possible pairs by the juggler's activity (calling the idle stock on the table a solidified unit) is its *entropy*.

The following picture of the physical nature of entropy is taken from the author's article in the November (1908) issue of the *Harvard Engineering Journal*:

"Every preparatory student understands entropy thoroughly, although he has never been taught to know it by name. For every school-boy knows well the value of organization for team-work upon the athletic field. He knows that, for good, effective sport, a given mass of men must be used, not as a single massive unit, nor yet as a number of independent individuals; but by subdivision, specialization and organization it must be animated into an organic whole.

"Indeed, without subdivision into at least two sides there can be no game at all; just as in mechanical energy there can be no energy until the mass present is subdivided into at least two portions, which mutually oppose and react upon each other. Similarly, the particular sort of energy called athletic antagonism onsists in the reaction occurring *between* two opposed sides. It loes not lie in either side alone, nor yet in the individuals alone—hough each of the latter possesses his own fund of internal nergy (due to his subdivision into a variety of organs, muscles, glands, etc.). For in a company of even the most stalwart thletes there could be embodied no other form of energy than nuscular, glandular, etc., exhibited in individual feats, unless here existed athletic organization into at least two contending arties.

"For in all the better sorts of athletic contests, such as base-all and foot-ball, there arises a quite distinct form of energy rom the organic energy of the individual. The Subdivision, pecialization, organization and interaction between players pplies not merely to the mass as a whole, but pervades each of

two 'sides.' One man trains to be a pitcher, another to be her, and a third to be short-stop. Each 'side' becomes itself organism, containing a form of energy which is quite distinct m the aggregate energy of the individual players. The two es coöperate competitively to the production of a game. The n on each side compete coöperatively to the advancement of t particular side of the game. There thus arises a more intri- e and effective form of contest than is possibly to be had from number of athletes whatever, trained to any degree of skill strength whatever, if each acts only as an individual. There arisen a new 'form' of energy: *team-work.*

"This subdivision and specialization into team-work consti- es athletic entropy. Correspondingly, athletic temperature is intensity and vigor of play of each individual player. Both cultivated by the coaches, as essential to good results; but y aid in those results in distinctly different ways. These ways ht be contrasted, for instance—if hearsay be accepted as true, the purpose—by saying that Harvard wins her victories by letic temperature, by brilliant individual play, whereas Yale s hers by athletic entropy, by a dogged devotion to team-work ch overbears in the long run.

"'Or a similar contrast might be borrowed from the field of t-engines. The gas-engine, with its tall and white-hot but htly entropied cycle, represents the acme of efficiency in ver-development from heat. Each molecule of its working- stance moves at tremendous radial speed, or temperature. the gas-engine has never been able to compete success- y, in the world's work, with the steam-engine; for the er's squat, cool and slow-moving cycle is so heavily entropied t the engine stays in the ring and continues to push d, long after the more delicate gas-engine has 'laid down on job.'"

This position is to be emphasized more as time passes. The portance of team-work is ever upon the increase. As popula- ns increase inventions multiply. A greater number of people now in daily communication with each other—and therefore ced to coöperate, whether they will or no—than a century could be reached in a month's travel. Life, in shop, market legislature, grows daily more multiplex and intricate. The d for the better and more patriotic understanding of human

relationships grows daily more urgent, while that for brilliant patriotism of individual deeds is on the wane.

All this is inextricably tied up with this question of entropy, or mass-pairing, or the quantity-factor of energy. In such a book as this no more than a suggestion of its broad significance and importance can be made. But to pass the occasion without even that would be a grave mistake. Our best records in American history are the victories won by national team-work. In 1864 it was the solidarity of the North, in an organized effort of each for all, which won over the more intense valor and superior tactics of the South, which lacked it. In 1898 all the credit we won was due to national entropy, and all our shame to a lack of it—to the false idea that intense and spectacular Rough Riding could accomplish what was desired, without the coöperation of every individual in the nation to the army's proper feeding and nursing.

The conclusions stated in the above pages are of fundamental importance. They may be summarized as follows:

1. *Temperature* is the intensity-factor of heat. It is the radial kinetic intensity of the mass-particles, large and small, of the molecule. It is proportional to $\frac{\text{velocity-squared}}{\text{total mass of molecule}}$, or mathematically, to $\Sigma\left(\frac{V^2}{M}\right)$. It is affected solely by changes of volume under pressure, contributing or abstracting radial motion only (or eccentricity of orbit) to the particles of the molecule.

2. *Entropy* is the extensity-factor, or quantity-factor, of heat. It is the extent of mass-pairing, or degree of subdivision, of the molecule, into separate particles which interact energetically. It is the variable proportion of the mass of each molecule effective in heat-motion. Mathematically, it is $\Sigma(MM)$.

3. *Labority* is the isentropic alteration of temperature, by elastic, or radial, work-performance or absorption. It is a variation of the intensity of thermal energy, or velocity of radial motion, under constancy of extensity, or degree of mass-pairing, or entropy.

4. *Thermogy* is the isothermal alteration of entropy, by inelastic, or tangential, work or heat absorption. It is a variation of the extensity of thermal energy, or entropy, or subdivision of the molecule, under constancy of its radial intensity, or temperature.

5. Both of these processes occur in apparent purity in nature. The first occurs in adiabatic expansion or compression—finding almost pure instance in the explosion of a steam-boiler, where lack of time excludes the conduction of heat to or from the outside. The second finds instance in the vaporization of water under constant pressure.

6. All other thermodynamic processes than these consist of combinations of the two—either produced simultaneously, by two outside causes acting at once, or occurring seriatim, one as the result of the other, within the internal equilibrium of the molecule. Instances of the latter are the isomorphic heating of water, where the temperature rises because the entropy has been increased; or the wire-drawing of steam, where the entropy increases because the temperature has fallen.

Instances of the simultaneous operation of two external influences occur in the compression or expansion of gases in actual cylinders. In compression the temperature is raised by work-performance and the entropy decreased by cooling, simultaneously. In expansion the temperature is dropped by work-performance and the entropy decreased by cooling. Every possible combination of thermogy, positive or negative, with labority, positive or negative, is known in power-house practise; and none of them can be explained consistently without this clear distinction between thermogy and labority, as independent, though sometimes connected, mechanical phenomena.

As possibly of further aid in understanding a difficult subject, the following parallel between temperature and entropy is given:

TEMPERATURE.	ENTROPY.
In Molecular	*Mechanics:*
Intensity of heat-energy, or space-and-motion factor.	Extensity of heat-energy, or quantity-factor of heat.
Radial activity.	Mass-pairing involved, maintained by tangential activity.
Degree of radial space and motion between mass-particles of molecule.	Extent of subdivision of molecule into mass-particles radially separated by space or motion.
Proportional to $e.\frac{U^2 \sin^2 \alpha}{M_1 + M_2} = e.\frac{c}{D}$.	Proportional to $\Sigma\, M_1 M_2$.
In Relation to	*External Bodies:*
Action normal to body's surface.	Action parallel to body's surface.
Elastic work-performance or absorption.	Inelastic work-absorption, or absorption or radiation of heat.
Isentropic labority.	Isothermal thermogy.

CHAPTER XIII.

The Energetic Cycle.

Hitherto the discussion has turned chiefly upon what energy is. It next becomes of importance to consider how it may be obtained for human consumption. The question is not one as to the sources from which energy may be derived. Much has been written upon this topic; yet, except for the need for endlessly repeating the lesson that the waves of the sea do not constitute a practicable source of supply, nor even the tides (except under most unusual circumstances), there is little to be said in this connection.

The question of prime interest is not: How may man acquire energy? It is, instead: How does nature do it?

On every hand, in every natural phenomenon, is seen an endless chain of energy-transformations. The one stock of energy possessed by nature is made available for the most intricate and endless variety of purposes and effects, merely by transformation. In the power-house one sees a little of this: The ownership of a store of black, dense, heavy, unchanging coal gives one permanence of potentiality for power. Upon demand, this may be converted into heat and light, in combustion. But the permanence is gone; transferability, and also evanescence, have taken its place.

The evanescence of flame-heat being usually too great, a fair degree of permanence is acquired, with portability retained, by transformation into steam-heat. This, upon demand, becomes mechanical motion. This, in turn, may become electricity. The electricity may then become chemical energy again, the form from which it started, in electrolysis; or light and heat again, as it was in the furnace; or an ether-wave carrying human intelligence across the midnight seas; or mere motion and heat again. Even in the human laboratory the transformations are startling enough. But in Dame Nature's they are far more so; they are stupendous, amazing and quite incomprehensible, except in their most general aspects.

To nature, therefore, the task of keeping all her vast and intricate processes in motion is not one of creation, or acquisition, of energy, from some other source. It is merely that of keeping what she always possesses in active transformation and circulation. Does nature wish to grow a world of fresh green verdure, some spring, or rear an entire human race in some twinkling of the universal eye? She does not transport a vegetation or a mankind from some other corner of the universe. She merely places between the sun and that unknown space into which it has been radiating its heat, throughout unrecorded time, a planet: an earth, watered and aired and fitted for its mission. The result follows. Without necromancy, as naturally and mechanically as a properly made motor starts when the current is turned on, a flood of verdure spreads over the face of the earth when spring comes, or a fauna of pterodactyls and eohippi develops into races of horses or men when the changes of season become sufficiently favorable.

It is quite similarly that man places his simple little contrivances in the way of the same vast currents of energetic flow in nature, that he may consummate his own tiny and short-sighted plans. He finds a reservoir of water up amid the hills, and sees its contents cascade toward the sea. In between the lake and sea he places his water-wheels, and derives power. He finds a red-hot fire, radiating heat into the lower realms of temperature. In between fire and refrigerator he places his steam-engines, and derives power.

The plan is a simple one to look at. But how does it work, in terms of the ideas as to heat and work which the preceding papers have outlined?

Take the simplest case first: that of the mill-pond, the tail-race and the old-fashioned overshot water-wheel between them.

The water in the mill-pond possesses energy because of its separation from the earth. It and the earth constitute two mass-portions, which are paired or opposed in a mutual reaction which we call force, or "weight," and which embodies energy. The two were pulled apart originally by sun-heat, a "supply of energy from without," and they tend to reunite as they reship their fund of energy upon its next journey in the world.

Man, however, cannot utilize an entire mill-pond of water at once. He therefore takes it a little at a time. A cubic foot of

water, or a ton, say, will be taken into the upper buckets of a water-wheel, to do work. This ton of water is *detached* from the natural supply, and introduced into the wheel; and in the performance of this process great care is taken to have no "free fall, friction or impact" occur, beyond what is unavoidable.

This simple process of detaching energy-bearing mass from the stock of it which nature brings to our hands, and its embodiment into the machine with which it is designed to develop power, is simplicity itself. Yet it is necessary to call attention to it in this special way, because it typifies a process which is an essential part of all energetic cycles, and which appears to be, in thermodynamic cycles, one of the most difficult of all processes for the student to understand.

When the stock of energy-bearing mass is once in the wheel it is permitted to lose its vertical separation from the earth, by falling, under resistance and control, to the tail-race level. It is there detached from the wheel and rejected into the tail-race.

According to the mathematics of engineering, if W be the weight of water taken into the wheel and h be the height of mill-pond above tail-race, the energy transformed by the fall is Wh. The cycle of events might be portrayed by a plain rectangular diagram, such as Fig. 10, in which the horizontal abscissae measure *weight* and the vertical ordinates measure *height*. If the point D should represent the weight and height of the empty buckets at the top of the wheel, DA the weight of the water taken in, OI_1 the height of the mill-pond above the sea-level and OI_2 the similar height of the tail-race, then the work done by the falling buckets would be measured by the rectangle I_1ABI_2 and that by the rising buckets by the rectangle I_1CDI_2. The net difference, or the rectangle DABC, would measure the net work done, or Wh. The efficiency with which the total available fall had been utilized would be $\frac{DABC}{DAX_1X_2}=\frac{I_1I_2}{OI_1}$

But under modern conditions the old-fashioned overshot or direct-gravity water-wheel has had to give place to the turbine. For handling large quantities of water under heads which often exceed the largest practicable diameter for wheels, the turbine is far superior to the older form. For very high heads and small quantities of water, such as occur in the mountains of the mining-regions, the Pelton wheel has been used in preference to

the submerged turbine. And as the different parts of the Pelton wheel are separated more distinctly to the eye, it may form a clearer illustration in the following discussion than the turbine.

For in both of these modern types of water-motor another cycle of energy-transformations than that just described has been interposed between the mill-pond and the tail-race. The water performs its vertical fall, not in the moving machine, but in a closed penstock. At the foot of this penstock its energy has been stored in the form of accumulated pressure. It is then chiefly "transient" energy, although a slight portion is stored elastically in the compression which the water has suffered during its fall.

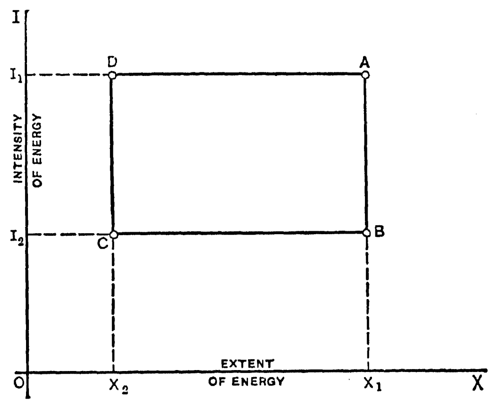

FIG. 10.

Before the water is admitted to the moving part of the wheel, this transient and resilient energy is converted into kinetic energy of jet, by the nozzle or guide-blades. It is in this form that it is received by the wheel proper, which is so designed as to be adapted to the reduction of velocity-relatively-to-the-earth by an alteration of direction of velocity-relatively-to-wheel, by means of curved vanes. The illustration of the manner of doing this, by means of parallelograms of velocity, is familiar to every technical student, and needs no reproduction here.

The energetic cycle which is thus performed by the nozzle and wheel in coöperation is quite similar to that of the overshot-wheel, in principle, and may be illustrated equally well by Fig. 10. Only, in this new use of Fig. 10 the vertical ordinates must

be accepted as measuring, not height nor head of static water-energy, but the velocity-squared of kinetic water-energy. The horizontal abscissae measure the quantity-factor, usually taken as mass, instead of weight, as before.

In this new case it is as true as it was before that the energy taken into the wheel is measured by the rectangle DAX_1X_2, that rejected to the tail-race is BCX_2X_1, and that converted into work is DABC. The efficiency of conversion, as before, is $\frac{DABC}{DAX_1X_2} = \frac{I_1I_2}{I_1O}$. The only difference is that now there are no "empty buckets" to be carried up and down, and so the vertical axis OI should coincide with CD.

In these two mechanical cycles, which are so familiar to all engineers that their description seems superfluous, there are visible all of the characteristics of the most obscure thermodynamic and other energetic cycles, if the positions taken in the preceding papers of this series be true. It is therefore important to observe carefully just what has happened therein.

In the first place, the question must be reviewed: What is potential hydraulic energy? What is kinetic hydraulic energy?

The equations which were given as exact answers to these questions, in Chapter I, are these:

$$\text{Potential energy} = c\ M_1M_2 \left(\frac{1}{S_0} - \frac{1}{S}\right) \tag{32}$$

$$\text{Kinetic energy} = \frac{1}{2} M_1M_2 \frac{V^2 - V_0^2}{M_1 + M_2} \tag{33}$$

In these expressions, it was pointed out, the factor M_1M_2 indicated the extent to which the total mass present was subdivided into mass-pairs capable of embodying energy, and was called the *extensity* of energy. The remainder of each expression indicated the degree of spacial or kinetic relationship which was embodied in these mass-pairs, to constitute them energetic pairs, and was called the *intensity* of energy.

The study of the general characteristics of mass, space and motion, as they appear all about us in nature, showed that each of these factors might be an independent variable. The extensity factor, for instance, varies in one direction by the *consolidation* of mass, and in the other by its *subdivision*. The

forces of gravitational attraction are everywhere and all the time tending to produce consolidation. The equally universal phenomena of motion, collision, impact and friction, engendering tangential motion and centrifugal force, are always tending to produce subdivision and comminution of matter.

The intensity-factors also vary in either direction, and under the same forces or closely allied ones. Gravitational attraction is always tending to increase the propinquity between masses, and likewise their velocities. The centrifugal forces developed by those velocities are always tending to decrease the velocities and the propinquity simultaneously.

Two Basic Processes of Mechanical Energetics. The two fundamental processes of mechanics, therefore, are:

1. *The variation of mass-pairing, or extensity of energy,* by the subdivision or consolidation of mass, *under constancy of intensity;*

2. *The variation of velocity or propinquity, or intensity of energy,* by the approach or separation, or by the acceleration or retardation of masses, *under constancy of mass-pairing.*

The second of these processes is apparently the more familiar. The performance of work by falling weights, or by hammers "slowing up" against anvils or by trains climbing grades by momentum, or by water accelerated in nozzles and retarded against moving vanes, are all familiar instances of variations in intensity of energy under *constancy of mass-pairing.*

But mixed in with and confused with these processes is the first named sort, which are distinctly contrasted with them in character. That is to say, before the weight can fall and perform work it must first be released, or "detached," from the earth, so as to form a mass-pair reacting with it. The hammer, before it can be swung on the anvil, must first be picked up. The water, before it can project against vane, must first be detached from earth and mill-pond. The railroad-train, before it can climb a grade by its own motion, must first be built and constituted a thing separate from the earth: ore must be mined and smelted, designs made, metal cast, wrought and machined, and the parts assembled into a complex whole. The complete structure must be equipped with water, fire, oil, etc. All this must precede the purely mechanical process of setting the train into motion fit to climb a grade.

In these examples it is clear that great differences in appearance occur in this one process of subdivision of the originally unit earth into a mass-pair capable of embodying energy. In the tripping of the drop or the picking up of the hammer it is so slight and incidental a thing that it is commonly overlooked as being a mechanical process at all. In the case of the manufacture of the railroad-train it is so complex and important that a whole string of factories is required for its performance.

The same thing is true of the different sorts of energetic cycles. The processes DA and BC are variations in the extensity of energy: The detachment of the working water from unity with the earth, in the first place, and its reconsolidation with the earth in the second. The processes AB and CD are variations in the intensity of the mass-pair, after its formation or destruction by the variations in extensity. In the hydraulic cycles these variations in extensity are commonly overlooked, as essential parts of the cycle; and yet they are difficult enough to perform properly. The water must be gotten into the overshot wheel with as little free fall, impact and friction as possible, and gotten out again in the same way. In the turbine or Pelton wheel the supply of motion-bearing water must be transferred to the vanes with as little impact and friction as possible, and the water must leave the vanes for the tail-race in the same way. The skill with which these extensity-varying processes are carried out often has as much to do with the efficiency of the wheel as has the skill with which the intensity-varying processes, by which the height or velocity of the water is reduced within the wheel, are performed—and in the former case usually much more.

It is in this careful way that the most familiar mechanical processes of the shop and the power-house must be analysed and understood, if they are to form a means for understanding the more obscure cycles of molecular mechanics. For, as the different forms of energy and methods of cyclical action are taken up, one after another, hardly any two will be similar in their external appearance. One will emphasize the DA-process, or the preparation of the mass-pair; as in the building of the railway-train and road. Another will emphasize the AB-process of dropping the intensity under control, as in the turbine water-wheel. Another, like the overshot water-wheel, will be excluded from practicability by its exaggeration of the CD, or empty-bucket, work, in

proportion to its net work DABC; while in the Pelton wheel the CD-work will be entirely absent. Yet the underlying principles are in every case the same. And as the study of thermodynamic cycles is reached this dissimilarity of appearance and identity of principle becomes most marked.

The first step toward these more obscure cycles is to note that the mathematical statements concerning the static and kinetic hydraulic cycles already given are not quite correct. That for the overshot water-wheel assumed that the weight of the water would be constant for all heights above the earth, which is not quite true. That for the Pelton wheel assumed that the kinetic energy was proportional to the square of the velocity, whereas the sum of the masses present must be inserted as a divisor. The true expressions for the intensity itself appeared in Equations 24 and 25.

In order, then, to have Fig. 10 become exactly and generally true, its axes must be understood as measuring, not weight and height, or mass and velocity-squared, respectively, but the true factors of intensity and extensity of energy, respectively. The ordinates must measure *intensity*, the abscissae *extensity*, of whatever form of energy is considered. That the mind may effect this transfer of ideas from the similes already employed, four mental substitutions from the approximate to the exact must be made. These are:

1. The substitution of the exact extensity-factor, or degree of *mass-pairing* or mass-product, for its approximate manifestation, *weight*.

2. The substitution of the exact spacial intensity-factor, or *propinquity* $= c\,\frac{1}{S}$, for its approximate manifestation, *height*.

3. The substitution of the exact kinetic intensity-factor, or *velocity-squared-divided-by-mass*, for its approximation, *velocity-squared*.

4. For heat-energy, the concept of Fig. 10 as portraying *both* of these intensity-factors simultaneously, for any given complex mass-system, instead of merely one alone.

This careful line of attack upon the mysteries of the thermodynamic cycles has been long and perhaps tedious; but the fruit is worth the trouble. The typical, or pure, thermodynamic cycle, the Carnot cycle, now appears as nothing more mysterious than

the cycles of the old-fashioned overshot and the newer Pelton water-wheel, in intricate combination; somewhat disguised by the number, minuteness and intricacy of the motion-bearing mass-pairs which supply the energy, it is true, but in no wise more mysterious in its principle of operation.

In 1824 Sadi Carnot, in his "Reflections upon the Motive Power of Heat," laid down these rules for the performance of a thermodynamic cycle with the maximum possible efficiency between any given temperature-limits:*

1. The heat must be taken in isothermally, at the temperature of supply;

2. The heat must then drop its temperature, performing work, isentropically;

3. The heat then rejected must pass to the refrigerator, or absorbent, isothermally, at the temperature of the refrigerator;

4. The remnant of heat still left must be raised to the temperature of the original supply isentropically, absorbing work.

The four processes thus defined are seen to constitute two pairs. Each pair consists of examples of one of the two thermodynamic processes which were defined, in Chapter XI, as the only basic ones. One of these was the isothermal variation of entropy. The other was the isentropic variation of temperature. *And these two fundamental thermal processes are identical with the two fundamental mechanical processes which were defined and numbered on page* 163.

That is to say, the four processes of the Carnot cycle may be represented by Fig. 10 with perfect accuracy, if the two axes be considered as measuring absolute temperature in the vertical case and entropy in the horizontal. Fig. 10 represented the kinetic mechanical cycle with equal accuracy when these two axes were considered as measuring intensity of space and motion in the vertical case, and extensity of subdivision of matter in the horizontal case. The only things needful to be kept in mind, in order to identify the two cycles completely, are three in number. In the first place, the distinction between radial and tangential mechanical action within the molecule must be understood.

*The language has been modernized into accord with the terms already made familiar to the reader in the preceding pages. Carnot's language was no less clear and explicit, except that he confused the ideas of Σ M and Σ MM—a confusion which is still widespread among teachers of energetics today.

Work can be performed only by *radial* mechanical or molecular energy.

In the second place, comes the simple, but confusing, question of *locality* in which the several processes are performed.

In the third place, any thermodynamic cycle, such as the Carnot, comprises *both* forms of mechanical cycle: the kinetic and the static, in its molecular action—just as a cannon-ball at the top of its trajectory will do work in virtue of both its height above the earth and its horizontal velocity too.

As to locality, in the hydraulic cycle, for instance, the cycle naturally starts with the process DA: the acquisition of energy-bearing mass. No thought is taken as to how nature may have prepared this natural supply of energy-bearing mass. Those processes lie far outside our water-wheels, and we overlook them. In the thermodynamic cycles, on the other hand, the preparation of the energy-bearing mass occurs within our power-houses, although not within the engines. The preparation of the steam in the boiler-room must be cared for and understood, as well as its work-performance in the engine-room. In the boiler-room the heat conducted through the boiler-shell acts tangentially upon the water-molecule's particles. At first, as the water is heating, the breaking up of the molecule by this action is quickly annulled by its compression into liquidity, by the superior pressure acting on the surface of the water. As each molecule attains sufficient internal whirling velocity, however, so that its centrifugal forces are able to overcome these external forces, the water-molecule pops into a steam-molecule. In order to give it its separateness, while at the same time conserving its same centrifugal force of whirling, or its "pressure," as we should say, with enormous increase in volume—a task similar to elevating a cannon-ball to the highest point of its trajectory while conserving its original muzzle-velocity—a large quantity of energy is absorbed, in latent form. The steam-molecule, thus charged with kinetic energy in the form of sensible heat, or temperature, and with space-energy in the form of volume and latent heat of vaporization, is then carried over to the engine, where it gives up both forms of energy, in part, to the piston.

The piston, it is true, is adapted only for the absorption of the kinetic energy, or temperature-heat; but while the kinetic *intensity* of the steam-heat is no greater than that of water of

equal temperature, the *extent* of kinetic energy, or the number and mass of the projectiles which are in a condition to impinge upon the piston has been vastly increased by the vaporization. A water-molecule engaged in work-performance is like a battleship firing one gun at a time. The steam-molecule is like the same ship firing a broadside. The intensity of projectile-energy (not the "intensity of fire," as artillerists use the term) is the same for one gun as for twenty of the same calibre. For the broadside the muzzle-velocity is no greater, but the effectiveness of fire—the "heft" of the blow against the enemy—is far greater.

The oft-heard but mistaken argument that steam-heat is inefficient for work-performance because of the latent heat in the exhaust is exactly parallel to the possible objection to broadsides as a waste of ammunition. If the enemy be present in form fit to absorb a broadside, it is vastly more efficient to employ broadsides than a dribble of single fire. The work is done promptly, powerfully and efficiently. And in building our steam-engines there is no difficulty in making the pistons fit to stand the broadsides. The powerful steady pressure of the latent heat of the steam is a vastly more useful laborer, taking the run of all conditions, than is the highly intense, but evanescent, pressure of the gas-engine cycle. The mere fact that latent heat costs something, as do broadsides, has little to do with the case. Many of our large gas-engine builders are to-day modifying their gas-producer accessories in their power-houses so that their engines may have more entropy to work with; though to tell them that this was what they are unconsciously doing, or that entropy were worthy of any engineer's serious study, would probably fill them with astonishment and disdain.*

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*Not only is it not true that latent heat is inimical to efficiency of work-performance, but it is true that no other form of heat is equally efficient. Thus, using water and steam as the illustrative working-substance, Fig. B shows the Rankine cycle of the ordinary steam-engine, using saturated steam, including the heating of the feed-water in the boiler. The heating of the feed-water is shown by the isomorphic curve AB, the vaporization of the hot water by the isothermal BC, the work-performance in the engine-cylinder by the isentropic CD, and the condensation in the condenser by the isothermal DA. The total area ABCDA measures the heat converted into work. Of this, that derived from the sensible heat of the water is the triangular area ABE; that derived from the latent heat of the steam is the rectangle BCDE. From this it is obvious that, for equal ranges of temperature and entropy, the efficiency of sensible or isomorphic heat for work-performance is *less than half* that of latent heat.

As the molecule expands against the piston, doing work, it loses kinetic energy under constancy of mass-pairing; that is to say, the fastest moving particles are slowed down, by rebound

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Indeed, could we build practicably an engine which would follow the cycle BCDE, *using only latent heat,* we should have it operating upon the Carnot cycle, the one cycle of maximum efficiency.

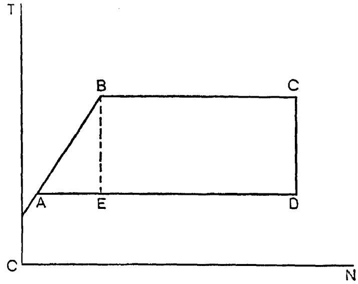

FIG. B.

Again, compare with the above the Beau de Rochas cycle of the ordinary gas-engine, as shown in Fig. C. Herein the total temperature-range, from A to C, is usually some 3600° F. Since the absolute temperature at A is about 600°, the best efficiency obtainable from the temperature-range available would be 3600÷4200=0.86. The efficiency of the Rankine cycle,

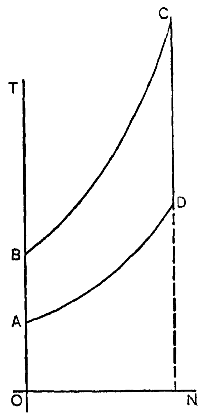

FIG. C.

under common ranges of pressure and temperature in condensing engines, or from about 600° to 800° absolute, would be similarly computed as nearly 0.25. Of this 25% available the steam-engine actually develops some three-fifths, or 15%. But of the 86% available for the gas-engine the latter seldom exceeds about three-tenths, or 26%. The difference is due entirely to the poor form of the gas-engine cycle. The trouble is that its heat is all isomorphic or sensible in form. In other words, were it practicable to build a heat-engine having the good points of the gas-engine, *but using latent heat instead of isomorphic heat,* its efficiency would be about twice that of the best standard gas-engines.

from a retreating piston, but without any *direct* tendency to diminish their number. That could be done, as was shown from the elementary mass-pair, only by a *tangential* retardation at apastron (the point of "contact" with the piston). The loss of *radial* component against the piston—which component is the only one which motion of the piston might affect, and which alone constitutes temperature—tends only to decrease the eccentricity of orbit and increase the periastron distance. The satellitic particles change their orbits from cometary forms, which whirl closely about the central nucleus and then shoot into space with great and almost radial velocity, to orbits like those of the outer planets: slow, remote and almost circular. The change is like that from a child's return-ball, shooting out and back on an elastic thread, to the same ball whirled about the hand on a string of fairly fixed length. And the "hot body," as the swarm of such molecules is called, becomes expanded, cooled, rarefied and of decreased expansive (or radial) pressure.

It happens, in the case of steam, that as this process goes on the form of molecular structure leads to a lack of internal equilibrium, so that, in order that the majority of the molecules may maintain the form described, a few are forced to recondense into water—partial condensation being a well-known accompaniment of the adiabatic expansion of steam. But this fact is merely an accidental characteristic of water. With ether, for instance, the exact opposite is true. Adiabatic expansion leads to superheat. Such facts as these, therefore, have no bearing upon the general idea that work-performance by heat is nothing more than a transfer of purely mechanical energy from the molecules to the piston.

For instance, in the nozzle of a steam-turbine of the de Laval type—and the nozzle is the only portion of this steam-turbine where thermodynamic action takes place—this mechanical explanation of adiabatic expansion is even more obvious. Here, instead of a retreating piston being opposed to the radially flying particles, nothing is opposed. The nozzle is merely a device for removing from the steam all resistant pressure in one particular direction, the forward one. The flying particles which happen to depart from their molecular nuclei in other directions than this meet the solid walls of the nozzle, and return with orbits unaltered. But those which happen to fly in the direction of the axis

of the nozzle find nothing to return them; and as they all possess hyperbolic orbits, they depart from home forever. Instead of giving up their radial velocity to a piston, they keep it. Only now it has become a linear velocity, not only in reference to the nucleus but also in reference to the walls of the nozzle. The swarm of molecules strings out into a long procession, like a body of troops defiling into column-formation; and although the cross-section has diminished, the volume has increased. The molecules find more "elbow-room." And while their original radial velocity remains unchanged, *relatively to each other* it has disappeared. Only the tangential component remains, giving to the orbits the same wide circularity which they had after collision with the retreating piston.

The impinging of this jet of molecules upon the curved vanes of the turbine-wheel, and the performance of work there by the loss of linear velocity, is a purely mechanical action, alike in principle in hydraulic and steam turbines. The expanded molecules pouring forth from the nozzle against the vane might just as well be so many microscopic but inflated foot-balls—originally fed into the boiler as so many solid wads of leather, and there inflated with compressed air by the energy of the furnace, and which have now escaped with a vigor of motion derived from their own internally stored energy—as expanded steam-molecules, so far as any thermodynamic action upon the vane is concerned. It is true that in actual practice thermodynamic action nearly always trespasses beyond the nozzle and modifies the vane-action in ways which cannot occur in hydraulic turbines. But this is a thing which the steam-turbine designer usually seeks to avoid, or to reduce to a minimum.

Cycle-efficiency. In any rectangular cycle, such as Fig. 10, the energy taken in is proportional to the initial intensity, I_1, whatever the width of the cycle horizontally. Usually, in nature, cycles are truly rectangular only when considered of infinitesimal width, dX. But since any cyclical area, however irregular in outline, can be divided into an infinite number of vertical strips of width dX between parallel sides, there is always a definite initial and final intensity of rectangular cyclical action, for each increment or decrement of energy handled.

The energy rejected by the cycle is similarly proportional to the intensity of rejection, I_2. Therefore the efficiency of a rec-

tangular, or perfect, cycle, or the proportion of the energy taken in which undergoes transformation, is given by the expression

$$F = \frac{I_1 - I_2}{I_1} \tag{34}$$

which is the fundamental equation for all pure energy-transformation.

In mechanical cycles these initial and final intensities are propinquities, or velocity-squared-divided-by-mass, as already defined. In thermodynamic cycles they are temperatures. In electro-mechanical cycles they are voltages. But whatever the form of energy may be the law stated in Equation 34 holds true.

Since the initial intensity, in natural action, can never be infinite, nor the final intensity zero, no cyclical action can ever convert *all* of any primary fund of energy into a secondary form. It will be shown later that energy-transformations are known between almost every two of the many known forms of energy; but in none of these many or diverse cases can the transformation be complete. Although it is commonly said that, whereas heat cannot be changed completely into work, yet work can be changed completely into heat, nevertheless this statement is not true, as will be shown later—except in a special, local and approximate sense which, while useful enough for certain purposes, has no place in generalities concerning energetic action. Indeed, we can acquire no proper understanding of energetic action until we understand that, not only is all of any energy-form *never* transformed into another form, but the difficulty of further continuing the transformation approaches infinity as the energy still awaiting transformation approaches zero.

Reversibility. Upon the surface of the earth, when dealing with solid or viscous bodies, only the purely vertical of these energetic processes are obviously "reversible." Therefore the rectangular cycle, of maximum efficiency, has usually been defined in terms of "reversibility." But in truth, speaking more generally of the principles of energetic action, not only do the vertical processes never occur in purity, or more than temporarily reversed, but the horizontal ones are equally reversible—as will be brought out in Chapter XV. As a temporary aid to the student energetic reversibility is a useful idea; but as a cosmic principle it must be handled with great care.

Such, in general, is the pure energetic cycle. It consists of four processes, viz:

I. The *subdivision of mass,* at a high degree of intensity. This subdivision may occur bodily, as in splitting off from the mill-pond the energy-bearing water which enters the water-wheel; or it may occur indirectly, as when a molecule of water within a steam-boiler is subdivided by tangential contact with the rapidly moving molecules of the hotter boiler-shell. In either case the work-performer (the water-wheel or steam-engine) receives mass which may be called "energy-bearing" because it is in a state of subdivision, with an intensity of either separation or of motion (or both, as in the case of steam) between every two particles formed by this subdivision.

II. The *loss of intensity* under *constancy of subdivision.* In water-wheel or engine-cylinder or steam-turbine nozzle alike, the number and size of mass-pairs at work is not affected by the performance of work. It is merely their intensity of relationship which is affected.

III. The *consolidation of mass* at a *lower degree of intensity,* the lowest which it is practicable to attain by work-performance. In the water-wheel this consists in dumping the now useless water into the tail-race. In the steam-engine it is the condensation of the steam in the condenser.

IV. The *raising of intensity* of the now partially consolidated mass, under *constancy of subdivision,* to the original intensity—which, because of the consolidation, takes less energy than was given out in the third process. This is the most obscure of all of the processes, because man usually leaves nature to do it for him, and she works in obscure and intricate ways.

The performance of these four essential processes requires the presence of four essential pieces of apparatus, viz:

I. *A supply of energy-bearing mass of high intensity of energy:* the mill-pond for the water-wheel, the steam-boiler and furnace for the steam-engine, the chemically stored energy in the explosive gases of a gas-engine charge. In the first case the mass is transferred bodily from source to machine; in the second case it is merely the subdivision which is transferred, by "contact." In the third case occurs no transfer at all, of either matter or entropy. The *chemical* subdivision between carbon, hydrogen

and oxygen is transformed directly into *thermal* subdivision, within the molecule, upon ignition.

II. *A device for dropping this intensity under control:* in an overshot wheel the vertical motion of the buckets; in a turbine the curvature and motion of the vanes; in a steam-engine the retreat of the piston under pressure; in the steam-turbine nozzle the open end and conical walls, with the curved vanes beyond.

III. *A device for absorbing the waste energy of lowered intensity:* in the water-wheel the tail-race; in the steam-engine the condenser, with its stream of cold water.

IV. *A means for the return of the rejected and consolidated mass to its original condition:* in the water-wheel the sunheat, clouds and rain; in the steam-engine the feed-pump and furnace-heat. It is this process which man, in all his prime-movers, leaves to nature to perform for him. Even in the case of the steam-boiler it is nature who is relied upon to furnish the coal to keep it hot.

CHAPTER XIV.

Reversed and Irregular Cycles.

The cycle which was described in the preceding paper, whether relying upon a water-wheel or a steam-engine for illustration, was considered always as passing through the four processes in an order which followed a clockwise direction of passage around the diagram. Such an order of procedure leads always to the drawing upon nature for a supply of energy of high intensity of one form, to the rejection of a remnant of that energy (with all the mass, or mass-pairing, which carried it) at a lower intensity, and the supply to the operator of work at a usefully high intensity.

It is obvious, therefore, that cyclical action is devoted to the *transformation* of energy, from one form which nature supplies to another which man desires. The first of these forms will be called the *primary* form and the other the *secondary* form. In the case of the overshot water-wheel the primary form is static gravitational energy, or space-energy, between mill-pond water and earth, while the secondary form is the transient energy of the wheel-shaft, which may be converted into any of many forms within the mill. In the hydraulic turbine-nozzle the primary form is static space-energy, as before, but the secondary form is kinetic water-earth energy of jet. In the vanes themselves of the hydraulic turbine the primary form is the kinetic water-earth energy received from the jet, and the secondary form is the transient energy of the wheel-shaft, as before. In the nozzle of the steam-turbine the primary form of energy is steam-heat of high temperature, involving both space and motion-energy between the particles of the molecules, as already described, while the secondary form is kinetic steam-earth energy of jet. In the steam-turbine vanes the primary form is kinetic mechanical energy of flying steam, while the secondary form is the transient energy of the turbine-shaft.

A moment's consideration will make it plain that there is a very close relationship between the individual links of any of

these chains of cycles. They are indissolubly linked. Thus, the water-wheel cycle cannot be performed unless there be a device for absorbing its power. Ordinarily this device was a pair of mill-stones, which absorbed it in friction. As this is too indefinite for our purpose, let it be supposed that the water-wheel drives a pump which draws water from the tail-race and discharges it into the mill-pond. Then, if the picture is to be complete, no water-wheel cycle could be drawn without drawing also a pump-cycle. And the pump-cycle, if friction is to be neglected for the moment, would be of equal area, or power, with the driving cycle.

But the pump-cycle must be portrayed in a *reversed,* or *counterclockwise* direction. It receives mass-pairing at a low intensity of energy, raises its intensity, discharges it into the pond (or consolidates it with the earth), and then goes back for more. Such a cycle would be shown by Fig. 10, if the diagram be traversed in the direction CBAD. The reversed cycle, therefore, is one in which the primary energy is received at low intensity and the secondary form discharged at high intensity.

It is merely a corollary of the law of the conservation of energy to state that for every clockwise or direct cycle which takes place, there must be performed a counterclockwise or reversed cycle of equal area, to absorb its energy. No water-wheel can operate without a pump or equivalent to absorb its power.

In portraying the action of any energetic machine, therefore, it is merely a matter of choice whether we portray the direct or the reversed cycle which occurs there. In the Pelton-wheel penstock and nozzle, for instance, there occurs a direct cycle of *static* hydraulic energy, which enters at high head and leaves at low; but there is simultaneously performed a reversed cycle of *kinetic* hydraulic energy, for the water enters at a low velocity and leaves at much higher speed. The cycles are like coiled springs. The unwinding of the gravitational cycle winds up the accelerative cycle; and the unwinding of the accelerative cycle, within the wheel, a moment after, winds up some equivalent cycle of a further form.

Thus nature works in an endless chain of transformations, by cyclical action, one thing giving up its strength that the next may live, and the next called upon a moment later to undergo

the same process of reproduction. In the power-house is seen quite a chain of such cyclical actions. The energy enters in the form of high-intensity chemical energy, in the coal-pile and atmosphere. This cycle unwinds in combustion, winding up simultaneously the cycle of thermal intensity, in high-temperature heat. This, in reality, unwinds again as its energy is radiated and conducted through the boiler-shell, though the fact cannot be seen unless it is analysed mechanically; and simultaneously it winds up a similar cycle, though a slower and more massive one, in the heat of the steam. This cycle unwinds again in the engine-cylinder, winding up the mechanical cycle of the engine-shaft. Or, if a steam-turbine be used, there are two transformations within the engine, one in the nozzle or guide-blades and the other in the vanes of the rotor.

So the energy might be followed upon its way, through electrical and other forms; and everywhere, so far as can be seen, the chain of unwinding and winding up continues endlessly.

In human affairs man is usually more interested in the unwinding, or direct, cycles; for he needs power more than he needs absorbents of power. But to a wide extent he also uses the latter. Pumps, elevators and refrigerating-machines all belong to this class. The cycle of the pump and the elevator is as simple, though reversed, as that of the overshot water-wheel, and needs no explanation. The cycle of the refrigerating-machine is much more obscure, however. For its details the reader must study the refrigerating-machines in detail. All that can be said here is that such a machine pumps low-temperature entropy (not heat) from the cold-storage room, or the water to be frozen, and discharges it at high temperature into any convenient waste, exactly as a pump picks up low-level water and discharges it at a higher level. The analogy is scientific and exact. It has already opened to our understanding wide fields of most useful progress in the arts, which await only a slight advance in our economic organization to make thoroughly practicable. We refer to the supply of heat for buildings upon a large scale by literally pumping up-temperature the low-temperature entropy which surrounds us during the winter months—perfectly good entropy, all of it; only a little too low on the temperature-scale for our use. Now we rely upon *heat* to heat our homes. Soon we shall reply upon *power* for that purpose; not by converting it into

heat, in friction, but in pumping out-door heat far enough up-temperature to be agreeable to human nerves.*

Irregular Cycles. So soon as attempt is made to carry out in practice any of the cycles described in connection with Fig. 10, it appears that it is impossible to do so, in purity. Nature never performs either of the two processes which were described as the basic energetic ones, the horizontal and vertical ones respectively, in purity. Each is always adulterated by the presence of the other to some degree.

To explain, let Fig. 11 represent an energetic field in which the sources and absorbents of energy are situated at the levels I_1 and I_2, respectively. These may be imagined as mill-pond and tail-race, if desired; but the diagram applies equally to any of the more obscure forms of energy. If, in such a field, it is desired to operate a direct cycle, such as DABC, it proves to be impossible, with exactness. In order to effect the transfer from the sources of supply to the machine, the energy must be taken in, in whole or in part, at levels somewhat below DA, as along *da*. Similarly, it must be discharged at levels somewhat higher than BC, as along *bc*. The area of the cycle *dabc* is therefore less than that of the cycle DABC, the cycle described by Carnot as the one of maximum efficiency.

Similarly, the attempt to carry out a reversed cycle, such as CBAD, between these intensity-levels develops the fact that the energy must be taken in at some level below CB, as along CGB, and discharged at some higher level than AD, as along AHD. This cycle therefore absorbs more power, as measured by its area CGBAHD, than is stored by it usefully, as measured by the area CBAD. Its efficiency, too, is below the maximum possible with rectangular cycles.

The irregular direct cycle *dabc* is like that of a water-wheel into which the water leaks while the buckets are still rising or after they have begun their fall, or out of which the water leaks before the fall is completed or after return has begun. The irregular reversed cycle CGBAHD is like a pump

*Tell a man who is spreading his hands before a blazing fire on a winter's night that what he is absorbing and enjoying is not temperature, but entropy, and the speaker would probably suffer. Nevertheless, it is true. Man takes almost enough bodily comfort, as well as industrial profit, out of this much abused, ignored and contemned drudge, entropy, to justify the proverbial saying that "ignorance is bliss."

which draws in at a level below that of supply, and discharges above the waste-level. It is like that of a hod-carrier who should be set to carrying bricks from the street-level to the third floor, but who took his hod into the cellar to drop the bricks into it, then carried it to the fourth floor, and then dumped the bricks back to the third floor.

From these considerations has been enunciated the general principle of energetics that the rectangular cycle of pure processes (as defined elsewhere in these papers) is the cycle of maximum efficiency. Carnot was the first to define this law, and he spoke in terms of thermodynamic cycles only; but the material for its application to mechanical cycles has existed ever since the work of Newton, and that for electrical applications ever since that of Faraday and Ohm.

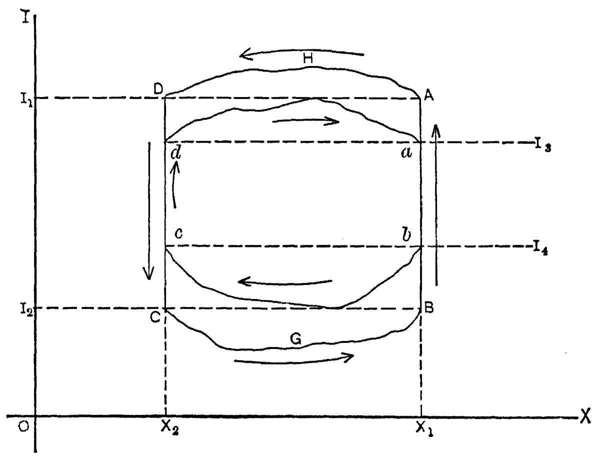

FIG. II.

If a direct cycle working between the limits I_1 and I_2 should be set to operate a reversed cycle (as always occurs in nature), it is plain that the direct cycle must take some irregular form, such as *dabc,* which is inscribed within the rectangle DABC. The reversed cycle which is operated from it must in turn be inscribed within the rectangle DABC. Therefore the limits of intensity between which the reversed cycle is finally effective must be continually lower and lower ones, such as I_3 and I_4, which lie between I_1 and I_2. It is this fact which has led to the

broad doctrine that the availability or intensity of the world's stock of energy is steadily declining.

Nevertheless, the statement is not true. It was based upon too obscure a form of cycle, the thermodynamic, for all its bearings to be clearly seen. So soon as it is referred to a form where all the details can be followed, as in mechanical energy, its falseness appears as unquestionable. For then the "degradation of availability" turns out to be based solely upon reference to a single form of energy—upon a form dictated solely by temporary human desire—and not upon any broad natural principle. Such a foundation is entirely too narrow to support a fundamental cosmic law. For what human beings preëminently desire seems to be rectilinear, or radial, motion. They have little use for tangential motion, except as it can be converted into rectilinear.

Thus, observe a steam-boat ploughing through the water. In the engines, in the steam within them, in the boat itself, and in the water surrounding the boat, is an intricate mixture of radial and tangential motions. Ultimately it all becomes virtually tangential motion within the water, in tiny eddies which constitute hydraulic resistance. These, later on, become still tinier molecular eddies called low-temperature heat. But one stage in the progress of the energy to this destination consists in a rectilinear, forward motion of the ship. Of all the motions present, this alone man esteems. He uses it as his measure of "efficiency." But nature esteems all motions equally, for she is able to reconvert the tiny eddies into rectilinear motion when she wishes to do so, as man cannot.

In the economy of nature the readjustment of equilibrium is being constantly made by means of these chains of direct and reversed cycles of energetic action. The currents of energy are fairly continuous, seldom starting or stopping abruptly. The portions of matter through which they must find their way, however, are limited in their mass and dimensions. Therefore each portion of mass, in order to perform its allotted task of energy-transformation, must act over and over again. This it does by the method of the cycle.

The ubiquity of these chains of cycles is too great for comprehension. Wherever energetic action occurs, there proceed these chains of cycles. We have twice referred to that visible

in the series of machines and processes incidental to the modern power-house. But is it realized that each tiny molecule, not to mention smaller portions of matter, in all this apparatus is itself constantly carrying on its own particular chain of cycles, peculiar to itself? It is the integration of these countless hordes of less than microscopic cycles, into something big enough for man to see, which constitutes that major chain of phenomena which characterizes modern power development and distribution.

But the power-house and its accessories are but a tiny instance of nature's broad dependence upon cyclical action. For a single instance, observe the interaction between vegetable and animal life. All vegetation is continuously operating a thermochemical cycle in a single direction. Drawing high-intensity radiant energy from the sun and low-intensity chemical energy from soil and air, in the form of the very stable chemical compounds, water and carbon-dioxid, respectively, it operates a direct thermal cycle to keep in operation a reversed chemical cycle. It maintains a low normal temperature automatically, so that we seek the "cool green shade" on summer-days; and it stores up chemicals of a higher intensity, in the form of starch, sugar and similar nutrients.

In apposition with these cycles, all animal life maintains their obverse. Animals absorb the starch and sugar and operate a direct chemical cycle in their degradation into carbon dioxid and moisture. They do this in order to keep in operation reversed mechanical and thermal cycles, resulting in animal motion and high-temperature animal heat.

Thus each half of animate existence here on earth both supports, and at the same time properly loads, controls and balances, the other. Without the coöperation of the other, neither could exist. Either starvation or apoplectic surfeit would ensue.

CHAPTER XV.

Thermal Equilibrium.

In summing up the general characteristics of mechanical energy, in Chapter VI, it was carefully pointed out that all energetic conditions of matter varied, not in one direction only, from an absolute zero, but in both directions, from a central or "mean energetic" condition. It was pointed out in detail how, from this central condition, each of the several factors of mechanical energy—force, velocity, space and even the mass-factor itself—vibrated in either direction indefinitely, yet limited elastically in stable equilibrium. For, as any factor proceeded away from the mean condition it engendered forces and phenomena which tended always to resist its further progress and to return it toward centricity. Thus, excessive velocity resulted in separation; but separation involved the storage of energy in space-form at the expense of velocity-form. Excessive separation, on the other hand, annulled the velocity which permitted it to exist, and invited the regain of velocity in a return into propinquity.

Thermal Equilibrium. If, now, attention be turned to Fig. 12, it will be plain that thermal energy shows every indication of following all the characteristics of mechanical energy, in its range from the unusually cold and solid conditions of matter, as at B, to the unusually hot and fluid conditions, as at H. The only difficulty in understanding the fact lies in the necessity of comprehending the unusually hot extreme in terms of the mechanical phenomena of the earth's surface; which, as has just been pointed out, concern themselves with matter in the extreme opposite condition, unusually hard and dense.

In the first place, the range of thermal conditions follows, for every substance, some such a curve as BCADEFGH. This curve exhibits stability of equilibrium at every point, except where it is interrupted by the fields of instability which we call fusion, vaporization and chemical dissociation respectively. Across all such gaps thermal conditions must jump abruptly. The curve

is asymptotic to the axis of absolute zero of temperature XX at the left. It can never reach it; and as it approaches it the increase in negative entropy, or solidity, becomes increasingly great with each step nearer.

The other limb of the curve, toward H, approaches increasingly the vertical direction. From knowledge yet available, which is comprised in Equation 30, it cannot be said explicitly that this limb is asymptotic to a vertical axis. But Equation 30, it must be remembered, is based upon too narrow a ground to be extrapolated into a general principle. It is based, first, upon an assumed constancy of the specific heat; yet of specific heats in the higher ranges of temperature we possess very meagre knowledge. It happens, it is true, that our most recent acquisitions in this direction point to an increasing specific heat for gases, as temperatures rise; and this would tend to maintain the obliquity of the curve at H. But the specific heat of water also rises with the temperature; yet this fact is only preliminary to a stage, the critical temperature, above which specific heats become very much smaller and the isomorphic curve much steeper.

Finally, reference must be had to the mathematical concept of the "perfect" gas, toward whose attributes gases tend as they rise in temperature; and this perfect gas, having no viscosity, could be represented upon Fig. 12 only by a straight vertical line. For all of these reasons, taken in connection with the fact that every other known energetic function becomes asymptotic at its either end, the conclusion cannot be escaped that the thermal diagram would also extend its upper end into a real, as well as an apparent, asymptosy, if its true form could be known; although it is impossible now to define either its true form of function or the lateral distance of its axis from any mean thermal condition.

Of these two axes to which thermal conditions are asymptotic, the first named or horizontal one, hitherto known as that of the "absolute zero of temperature," will be referred to hereinafter as the axis of *absolute solidity of matter,* where exist no fluidity, no elasticity, no expansivity and no translucence. To these characteristics might be added no temperature, infinitely negative entropy, no volume, and infinite density. But whereas the latter list is meaningless to us, we have an idea that we know what the former qualities signify; for are they not the ordinary

attributes of solid matter? Yet the method of approach to the statement just given was chosen in order to make it clear that this axis of absolute solidity defines mathematically a condition which can never be reached in nature. Its characteristics are those which matter never can exhibit.

Of the two axes of asymptosy, the other or vertical axis is that of the so-called "perfect," or, as it will be called hereinafter, the *absolute, gas.* In this condition matter possesses perfect fluidity or no viscosity, perfect elasticity or no disgregation-work, and perfect translucence. Incidentally it must possess either infinite temperature or infinite volume, or both, or its density must be zero. Viewed mechanically, its orbits must be all hyperbolic and none elliptic; its mass must be all satellitic and none nuclear. This, too, is a condition which may be mathematically defined—and such a mathematically defined limit is most useful to the understanding—but it is a condition which, however closely approached, may never be reached in nature.

Now this so-called perfect gas—as if anything which the Supreme Intelligence had seen fit to exclude sweepingly from the universe could be called "perfect"—is so impossible and unnatural a thing that the writer has carefully excluded it from all of his teaching. But it has been used so widely, by other teachers, as the base and explanation of all thermodynamic action, that it must be mentioned here to the extent of putting it in its proper place.

That is to say, there exists on one side of all energetic action (at the lower left-hand of Fig. 12) an unattainable limit of deficit of internal energy, when all internal motion would be purely tangential, or circular, and the pressure zero; which condition would be attainable, if at all, only when the temperature were absolutely zero and the quantity-factor of heat infinitely negative; in which condition, if ever attainable, the substance would constitute a "perfect"—or, as I prefer to state it, an "absolute"—*solid.* Such a state of affairs would be exhibited by the point B, Fig. 12, if pushed far enough to the left (that is, to an infinite distance) to bring it into coincidence with the axis XX.

On the other side of all energetic action (but not on the *opposite* side) there exists an unattainable limit of surplus of internal energy, when all internal motion would be purely radial,

or rectilinear, and none tangential; and where the viscosity would be zero; which condition would be attainable, if at all, only when the temperature were infinite and the quantity-factor of heat at its maximum, as shown by the axis NM. Such a state of affairs would be exhibited by the point H of Fig. 12, if pushed far enough up (that is, to an infinite distance) to bring it into coincidence with the axis NM to which GH is asymptotic. This axis is therefore that of the so-called "perfect"—or, as I prefer to call it, the "absolute"—gas.

Between these two unattainable extremes, or purely mathematical limits, all natural action, thermodynamic or otherwise, occurs—surging back and forth in stable equilibrium about some central mean energetic or thermal condition. In so far as it gains approach to the vertical axis of zero solidity, or gaseous perfection, it gains fluidity, elasticity, expansivity and adaptability for work-performance. And in this sense it is true that, to the extent to which matter possesses those qualities which, if existing alone, would constitute it a "perfect" gas, it exhibits faculty for thermodynamic labority. But equally true is it that in so far as it gains approach to the horizontal axis of zero gaseousness, or perfection of solidity, it gains faculty for impact, friction and gravitational work-performance; in other words—to coin a parallel term—for dynamothermal thermogy. And these two contrasted faculties, it seems to me, are of equal importance and deserve equal prominence in thermodynamic discussion.

Neither limit of thermal attributes can matter ever reach. Yet each of these limits has its proper use, as a base of reference. Some writers on thermodynamics, if not all, have preferred to enter the topic from the limit properly called the "absolute" gas as their base of reference. To this plan, so carried out that the student sees the true relation between base of reference and all natural action, there can be little objection.

The writer has much preferred, however, to enter the topic from the other limit, that of the "absolute" solid, by studying first the elements of energetics as they would occur between two mass-portions each of which is apparently a solid unit. This is natural because, in the first place, the growing boy has dealt almost entirely with solids, in work and play, and all his concepts of mechanical action are based thereon. In the second place, the mechanics of solids is already reduced to an exact mathe-

matical science, which is supposed to be taught to every student of engineering before he meets the problems of thermodynamics. So that the writer regards this mode of entrance to thermodynamics as far preferable to that via the "perfect" or "absolute" gas.

The one plan which deserves unlimited condemnation, however, is to open the study of thermodynamics with the concept of the absolute gas, using actual gases as "impure" illustrations, and to leave the student with the idea that what likeness to the absolute gas can be found in nature is alone thermodynamic action. All natural action is thermodynamic, to whatever extent heat takes part. To leave the student with his concept of natural action thus equipped with a roof, in the form of a knowledge of the gaseous side of thermodynamics, but with no foundation or adequate frame-work in the way of a corresponding knowledge of the solid and liquid aspects of the science, is what is being done by every teacher who thus misuses the mathematical concept of the so-called "perfect" gas.*

This fundamental fact should be one of the first taught the student of natural science. These two axes define the boundaries

*The following language concerning this situation is taken from another article by the writer: "We know of no gas so hot or so rarefied and so 'perfect' that it loses all viscosity, or is perfectly elastic and free from disgregation-work. We have no reason to think that any such a substance could ever exist. It is unobjectionable to refer, upon occasion, to the mathematical hypotheses of the 'perfect gas' or the 'perfect solid' as aids in argument, as has just been done. But with these references should always go the explanation that these two hypotheses constitute impossible, supernatural extremes, between which vibrate all known natural conditions of matter; and science should have just as little to do with the supernatural as possible.

"But when reliance upon the perfect-gas hypothesis goes so far as to make it the center and foundation—the immediate beginning, rather than the remote and unattainable horizon—of all thermodynamic study, as is now very widely done, the writer feels impelled to rise, in the name of nature and common sense, to denounce the practice. His experience in teaching thermodynamics has found this method so universally confusing to the engineering student, who above all others needs acquaintance with nature rather than with disembodied and supernatural hypotheses, and so belittling to the dignity of the instructor posing as a Guide to the Truth, that he can express his feelings only by a free quotation from Mr. Burgess's 'Purple Cow'—with apologies to the Cow, as well as to Mr. Burgess:

'I never saw a Perfect Gas.
I never hope to see one.
But I can tell you, as we pass,
I'd rather *see* than *be* one.'"

of the territory within which occurs all natural action. They are dead-lines which not only matter, but even thought, may not touch without ceasing to be. A great deal of physics may be taught, it is true, without mention of either the absolute gas or the absolute solid. But if either of them *is* mentioned at all—and their use is common in teaching even elementary physics—both should be mentioned together, and both should be displayed as supernatural concepts. Here again, neither Siamese twin should be presented with the message that the other had not yet been separated. It is as absurd to mention to the student the absolute zero of temperature without parallel mention of the perfect gas, or *vice versa,* as it is to mention the force of gravitation without its inseparable companion, centrifugal force, or to describe a chemist's balance as a pan suspended from a rod into which substance may be put for weighing, but with no mention of the other arm and pan into which the weights are put.

Indeed, the very description of matter, to the student fit for generalities at all, should be made in such a way as to show that solidity and expansive fluidity of matter are purely relative terms—that all matter is partly solid and partly gaseous, that what we call "solid" matter is merely matter more solid than gaseous; that what we call "gas" is merely matter more gaseous than solid; and that nowhere in nature occurs matter which is either wholly solid or wholly gaseous.

Although water alone has been selected for illustration in Fig. 12, because of its familiarity in solid, liquid and gaseous states, yet these general conclusions as to the characteristics of thermal condition and action of mass apply equally to all sorts of matter. All that is needed in order to bring the thermal characteristics of any substance into the form shown is to alter suitably the scales of temperature and entropy. Thus, for the hydrogen-oxygen mixture whose curve is shown as hRGH, all that is necessary is to expand sufficiently the temperature-scale and reduce the entropy-scale, and its field of fusion and vaporization would appear upon the diagram in much the same form as that shown for water. Similarly, the curves for any of the calcium or silicon compounds, which remain refractory solids at all ordinary temperatures, might be brought into similitude to the water-steam curve merely by reducing the temperature-scale and expanding the entropy-scale. For all known substances pass

through the solid, liquid and gaseous states, under suitable conditions. Fig. 12, representing, as it does, not only these three states, but the processes of transition between them, and also chemical transformation, may be regarded as displaying the thermal action of all known substances—so far as they connect with work on the one hand and with chemical energy on the other.

With this in mind, let the curve be traversed from B to H, as before, but with thermal conditions treated as micro-mechanical ones.

At B the substance is a "solid." Energetically speaking, a solid is a portion of mass all of whose parts act as one, in their reaction with external forces. If one portion moves, all the rest move simultaneously an equal distance. Between the particles of the system there is no motion at all, except tangential motion. The tangential motion, however, must be plentiful.

The necessity for this tangential motion can best be understood by reference to the story of how St. Patrick cleared Ireland of its snakes: Of how he invited all the toads and snakes to a banquet, at which no food was provided. Thus the snakes were led to devour all the toads, and then to fight and devour each other. And the end of all this was that there were left, finally, but two snakes, containing all the rest; and then these two each got the other's tail in his mouth and swallowed and swallowed, until nothing at all was left!

For the force of gravitation, it is plain, if left to itself, would do just this. It would deprive matter of its occupancy of space. For we know of no such thing as an absolute or "perfect" density. Every dense portion of matter which has yet been examined has proven to be merely an association of portions still more dense, the further characteristics of which portions were as yet obscure to us. Mass and space are apparently independent. The mutual gravitation of mass increases with the square of the propinquity, with no known limit. Mass, once drawn into propinquity by gravitation, is ever urged on into even greater concentration.

Here is obviously unstable equilibrium. If gravitation were the only cosmic force the universe would quickly become a single geometric point, of infinite density. The universe can be imagined as continuously existent at all only by the presence of

universal tangential motion, developing centrifugal force sufficient to counterbalance gravitation. The more matter concentrates the greater must be these tangential velocities and centrifugal forces. The greater, too, must be the rigidity of each tiny whirling mass-pair. But the student should be cautioned at every point against any thought of any absolute or final limit to the smallness, density, speed and rigidity of such "particles" of matter. He should be reminded that it is just as foolish and needless to speak or think of any "ultimate" or "indivisible" particle of matter as it would be to speak—as the ancients always did—of any solid fundament below the universe, upon which it rested, or of any rigid limits to space, beyond which the universe could not extend and where existed—what? Neither beginnings nor ends of anything, whether of time, space, density, solidity, elasticity, intensity of energy or aught else, either exist in nature or can be comprehended by man.

The student need not be told this. But his teachers very much need to be reminded to refrain from suggesting to him the opposite idea.

Therefore, to return to the cold facts of Fig. 12, however far the point of natural condition B may be imagined as departing to the left, approaching the condition of absolute solidity more and more slowly, it must still be always within a finite distance of its mean energetic condition. While all cold, hard solids possess very great solidity, rigidity, density and inelasticity, they do not altogether lack fluidity, ductility, space, expansivity and elasticity. They all occur at very low temperatures, relatively to their liquids and gases; but they always possess some temperature. Heat can always be extracted further from them. In short, their condition is always transmutable into liquids and gases by processes of finite dimensions. We therefore surmise that, while the greater portion of the mass of such bodies revolves, in exceedingly dense particles, with almost circular motion at exceedingly small radius, and with very great rapidity and rigidity, yet there exists not only some slight eccentricity to these orbits, but a minor number of mass-particles revolve about highly eccentric, or even hyperbolic, orbits. The tiny, rigid, tangential pairs arrange themselves into some form of stable equilibrium, called molecular and crystalline. Between and about and out from them shoot the minority of satellitic projectiles, exerting some slight expansive vapor-

pressure and affording some slight degree of elasticity and some slight manifestation of temperature. At human temperatures many crystalline "solids" exhibit considerable fluidity, ductility, elasticity, etc.; but when reduced to the temperature of boiling hydrogen these same substances become rigid and brittle in the extreme.

It is such "solids" as these which, by their relationship in visible distances of separation and visible velocities of motion, embody what we call "mechanical" energy. Here on the surface of the earth we see these solids related in motions, all of which are below the lower critical velocity, and which end promptly in collision with the earth and with each other. They are therefore inseparably associated with impact and friction; that is, with thermogy. Their energetic history ends always in this. Every bit of mechanical energy aroused here on the earth's surface, whether by our artificial heat-engines, or by the sun-heat acting upon wind and water, or by the moon acting upon the oceanic tides, ends its existence, in a surprisingly short time, in transformation into heat. The region of mechanical energy is the region of solidity, opacity, brittleness, inelasticity and density.

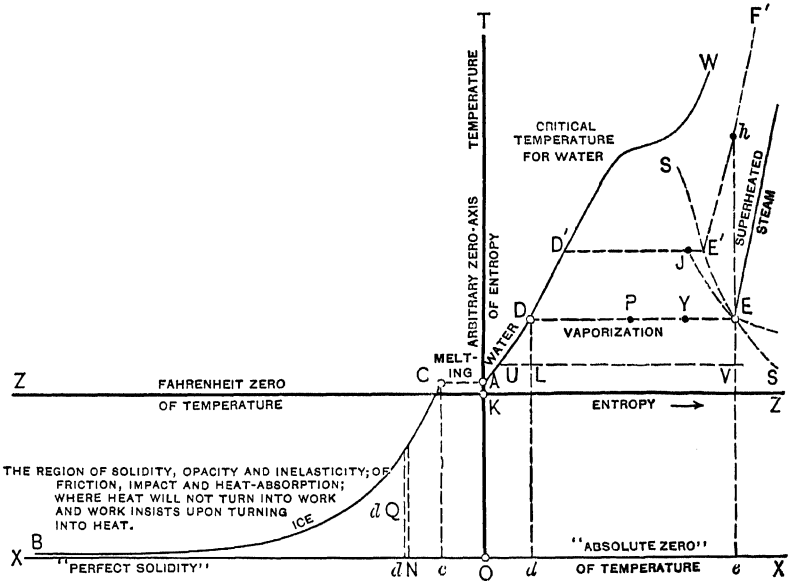

FIG. 12A.

It is the region of impact, friction and thermogy. There all "work" insists upon turning into heat, and heat will not turn into work. It is the region displayed at the lower left-hand of Fig. 12. It is a region which belongs especially to the solidified planets of the universe, of which our earth is one. It is particularly the subject of what we call the "applied mechanics of engineering"; for while engineering does not deal largely with ice, which forms the illustration in Fig. 12, yet it does deal, almost exclusively, with substances, such as iron, stone and wood, which are as far below

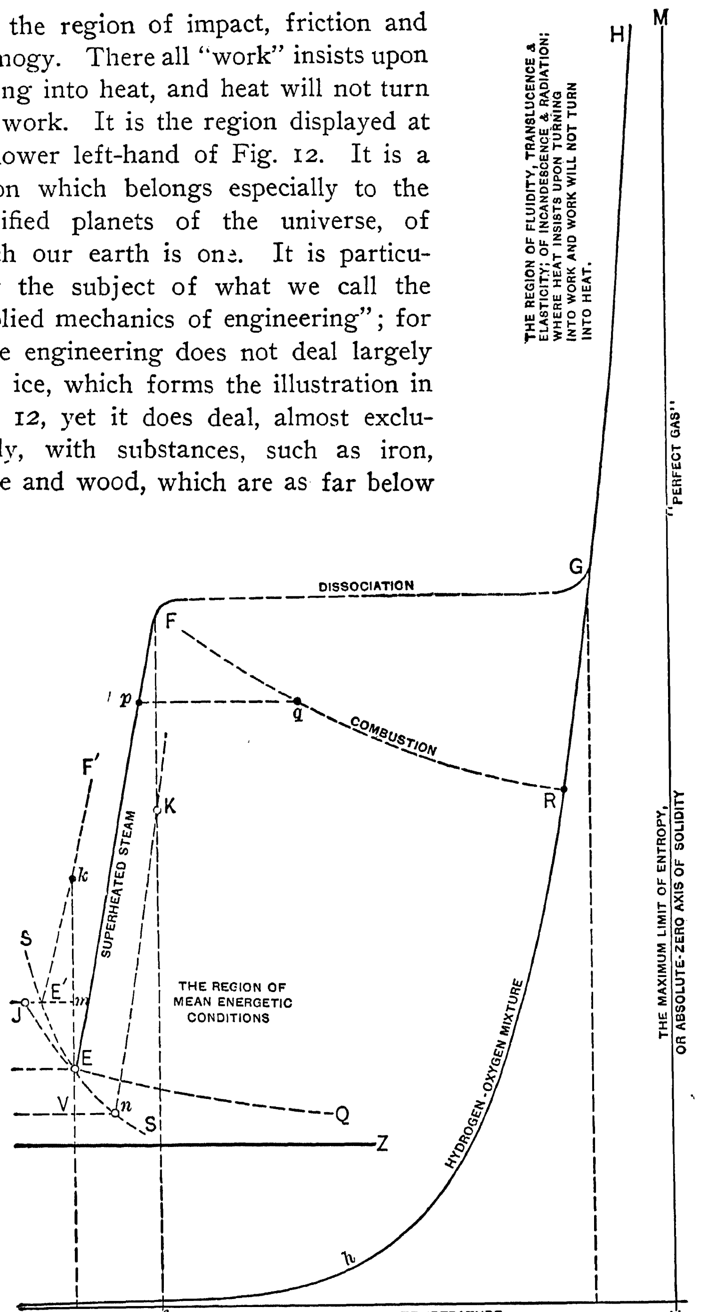

FIG. 12B.

their mean energetic conditions as ice is below boiling water.

Yet the prime characteristic of this region is, as just stated, the thermogic formation of heat. Being the region of deficits of heat, entropy and temperature, the prime result of its activities is to make good these deficits, in all three lines. Its activities being of an extreme energetic nature, they show every disposition to get back toward the mean energetic condition. They not only tend constantly to produce entropy—which inspection of Fig. 12 shows that they need even more than temperature—but they produce it under such conditions (namely, a deficit of internal expansive, and a surplus of external, pressure) that the entropy-growth is promptly converted into temperature-rise. Whatever causes may be offered in explanation of how these solid bodies ever got into this extreme condition portrayed at B, Fig. 12, there can be no doubt as to their unwavering tendency to return toward centrality, along the path BCADE.

The impact and friction constantly occurring between such solid bodies develops entropy in collision, as has been described, by smashing them into finer and finer fragments, until collision is no longer possible. Simultaneously, to these fragments are imparted sufficient velocities of *tangential* motion to enable them to remain separated fragments, without falling together in obedience to mutual gravitation. This process we call the creation of heat. So long as the substance is still solid, the result of this increase in tangential energy is to arouse a resistance from without, which compresses the widened circular orbits until some portion of the energy is squeezed into a *radial,* or satellitic, form. This increase in radial energy is perceptible from without, whereas the tangential energy was not, and leads us to observe directly that the "temperature" of the body has risen. It is much more indirectly and slowly that we have come to a knowledge of the latent, or tangential, increase in entropy which preceded this.

In the case of solids below the fusion-point the internal tendency to increase of volume and fluidity, with increase of entropy, is resisted by external forces which we are forced to call "crystalline," in lieu of fully understanding them. In the case of liquids below their boiling-points, however, the internal expansive forces are resisted by an external fluid-pressure which is familiar to all; and the action of this external pressure,

in squeezing the entropy-gains into temperature-gains, is exactly that of the piston of the air-compressor or the cushioning steam-engine, in squeezing volume into pressure and temperature.

When such points as C, or D, or F are reached, however, the effects of thermogy, in developing entropy, volume and elasticity, is no longer opposed effectively by the external pressure. The processes of fusion, vaporization and dissociation, respectively, occur in purity, as developments of entropy, latent heat, volume and elasticity at constant pressure—as a development of subdivision and disgregation of matter, and of radius of tangential motion, without any increase in radial kinetic energy.

It thus becomes plain how matter which possesses a deficit of radial energy, of satellitic mass, of volume, expansive pressure and elasticity, with a surplus of tangential velocity, density and rigidity, begets naturally those things which it lacks and rids itself of those things which are in surfeit, as it passes along its path of thermal conditions, BCADEFGH.

It is now to be noted most carefully that the motive power of those peculiarities, which started it along this path, dies out as it proceeds. That is to say, as it gains radial energy, temperature and satellitic mass, and more particularly, as it gains entropy, volume and elasticity, *the effectiveness of impact and friction for heat-development die out.* The causes which forced the substance toward its mean energetic condition lose their force as that condition is reached, and counter-tendencies begin to prevail. Thermal or other energetic conditions, like pendulum-bobs, lose their motive power as they approach their central positions.

This does not occur abruptly or completely. Gradually rigidity, density, inelasticity and opacity fade away, but they never completely disappear. We know of no substance which does not possess these qualities to some slight degree, which does not carry on some slight thermogic action. Even the most rarefied of gases possess some slight viscosity and absorption.

With these diminutions in impact and friction must be considered also thermal conductivity and the absorption of radiation. Of these, in detail, we know nothing, except that their results are identical with those of impact and friction. Here too, in the progress along the thermal path, the likeness again

appears. Speaking generally, it is the forms of matter which are most rigid and dense, and most subject to impact and friction, which possess the highest rates of surface-absorption of heat and its conductivity throughout the mass. To this rule there are many minor exceptions, but they are local and insignificant. The broad fact is as stated. While the contrast between solids and liquids in these matters is not marked, that between liquids and gases is so. The gases are most difficult to impart heat to. Air is much harder to heat than is saturated or slightly superheated steam, and steam is much harder than water. As to solids, the difficulty in heating them does not lie in any lack of absorptivity, but in bringing ordinary thermal media, which are usually liquid or solid themselves, into effective contact. Therefore, as matter proceeds along the thermal path from the solid extreme toward the gaseous, it loses first its liability to impact and friction, and secondly its liability to the receipt of entropy by conduction or absorption.

Moreover, the liability to thermogy does not depend solely upon the condition of the body's surface. The rate of heat-transfer depends also upon the difference in temperature between contributing and receiving body. When the recipient is a very cold solid, nearly all other bodies are warmer than it, and very much warmer, and contribute heat rapidly. But as the substance grows warmer it leaves behind it, one by one, those neighbors which were recently able to impart heat to it, and in turn begins to impart heat to them. Until finally, when it has become unusually hot, as toward the H-extreme of the thermal path, it is only the semi-occasional body which is still warmer than it, and so able to contribute heat to it.

At this end of the path, too, rigidity, viscosity, impact and friction have almost entirely disappeared. In their place now appears an excess of volume, expansive pressure, fluidity, elasticity and often incandescence. Here work will no longer turn into heat, by thermogy, to appreciable degree. Instead, *labority* has become the prevailing phenomenon. The heat insists upon turning into work. And what heat cannot find conditions suitable for its conversion into work, insists upon radiating and conducting itself away to colder bodies.

Viewed constructively, the molecule now appears to consist almost entirely of minutely subdivided particles moving in

highly eccentric and chiefly hyperbolic orbit. There still exist nuclei, but they embody a minor portion of the mass. There still exists tangential motion, but it is now an insignificant base for the prevailing superpermanency of radial energy.

It is because of this paucity of tangential motion that it is so difficult to impart further heat to the gas, by thermogy; for impact, friction, conduction and the absorption of radiation, all must occur through the tangential components of motion, although they may find final expression in radial motion. It is because of the superabundance of radial energy, on the other hand, that it is so easy to raise the temperature of a gaseous body by compression; for compression acts directly upon the radial component, being transformed into tangential motion only secondarily, under the squeezing of the nuclei into greater propinquity. Only, it is to be noted, as the temperature of a gas increases so also does its pressure, other things being equal; and as the condition of the substance becomes extreme in the H-direction it becomes increasingly difficult to find a force capable of compressing the gas, although it may become increasingly fluid and elastic and capable of compression as it goes.

Energetic Gravitation. There thus become visible in the thermal field two fundamental gravitational tendencies. One of these is for *all* substances, and particularly those to the left of the mean energetic condition, to pass horizontally to the right, by thermogy. The other is for *all* substances, but particularly those above the mean energetic condition, to pass vertically downward in temperature, by labority. Roughly speaking—and perhaps accurately too—the tendency to fall in vertical intensity (or, in this case, in temperature) is proportional to the intensity itself, or the distance from the horizontal axis of absolute zero of temperature. In saying this we have not in mind the force with which it tends to fall, but the chances of that force being able to prevail.

Similarly, the tendency of energy to increase in quantity-factor (in this case entropy, although the statement applies broadly to the mass-pairing or quantity-factor of any form of energy), by "collision," or resistance encountered in motion, is proportional to the distance of the energetic condition from the vertical axis of absolute zero of solidity, or from the condition of the perfect gas, at the right. In this we see that we have

unconsciously formed the habit of speaking of entropy with the positive and negative signs reversed from what they naturally should be. Just as, in speaking of space-energy, in order to be consistent we have been forced to substitute the idea of *propinquity,* or lack of space, for space itself, as a measure of intensity, so, in speaking of thermal energy, in order to be consistent, we should always refer to entropic changes in terms of lack of *solidity*. Solidity tends to decrease, in the universal thermogic aspect of thermal phenomena, just as persistently and uniformly as temperature tends to decrease in their work-performing or laborious aspects. Had the idea of entropy and thermogic tendencies been originally thus handed out to the student right end foremost, as a simple universal tendency of solidity to disappear in impact and friction, quite similar to the tendency of temperature to disappear in work-performance, the subject would never have become so enshrouded with mystery and awe as it is at present. But now, so strong is habit, it will be a long and difficult task before the customary algebraic signs of entropy will be reversed.

These two universal tendencies are the *energetic gravitations*. Each prevails as constantly as does the Newtonian centripetal gravitation of mass and centrifugal gravitation of motion. In mechanical energy these tendencies are, as to its intensity, that of either unusual velocity, unusual space or unusual propinquity to decrease to a minimum; as to extensity, that of the mass-pairing of matter to proceed to a maximum, in its further and further subdivision (or, in other words, the tendency of the solidity of all matter toward a minimum). In thermal energy these tendencies are that of temperature to decrease and that of entropy to increase. All thermal phenomena are the result of a balance between these two gravitational tendencies; for the gravitations act always in opposition, in a counterbalance between each other and with outside conditions. Sometimes one prevails over the other, sometimes the other over the one; sometimes both together prevail over outside forces, sometimes both are overcome thereby.

If Fig. 12 be held up by its upper left-hand corner, the thermal path BCADEFGH will then appear as the somewhat irregular path of a pendulum-bob, swinging from the point of suspension. Just as the bob of a real pendulum is impelled

toward its central position from either side, whichever it may chance to reach, so is the thermal condition of matter impelled always toward its mean energetic condition, from either thermal extreme which it may chance to attain, by these two vast gravitational tendencies. Across this great arc of physical condition all thermal phenomena are constantly swinging back and forth, as in a gigantic pendulum. For the ranges of the arc are indeed gigantic. From the coldest density of solid matter lost to view in interstellar space, on the one hand, to the highest temperatures of the incandescent suns and the extreme tenuity of the luminous nebulae and comets' tails, on the other, thermal happenings are constantly swinging. And no heat-action on the surface of the earth can be understood without reference to both of these extremes.

What combinations of conditions in nature's vast laboratory may have led to the existence of suns and nebulae on the one hand, and of remote dark stars and cold meteorites on the other, is a great and open question. Its answer has nothing to do with the present argument. The fact remains unquestionably fact that, these extremes of heat-condition once in existence, their tendencies must be as described.

Yet the conclusions therefrom must not be too hasty. Carnot and others before him noticed the downward tendency of temperature. Clausius noted the outward tendency of entropy—or the downward tendency of solidity. It was Zeuner who deduced therefrom the quite unwarranted conclusion that the entropy of the world was not only tending, but was actually moving, toward a maximum. It was Lord Kelvin who put these ideas together into the doctrine of the steady "degradation" and loss of availability of energy.

These mistaken conclusions depend upon seeing these two tendencies as working always and everywhere *together* to a single end, viz: to the reduction of all energy to heat, and all heat to a maximum of entropy and a zero of temperature-difference. The facts are just the opposite of these. The two tendencies are *always opposed*. Entropy cannot be increased unduly except by processes which simultaneously increase the intensity of temperature and its availability for labority. Therefore entropy itself constitutes a factor in availability. For thermal radiation and work-performance it becomes available when extreme in

the positive direction. For thermogy and mechanical energy it becomes available when in an extreme condition which we, mistakenly, call "negative" (as at B, Fig. 12). For matter in this extreme condition embodies as great an intensity, or availability, of *mechanical* energy as that in the extreme condition at H embodies intensity and availability of *thermal* energy. Indeed, the B conditions are attained by matter only when it becomes so remotely isolated from its fellows, in the heavens, that any occurrence of thermogic collision at all must develop so much heat as to transfer instantaneously all the matter involved into an equally extreme condition of temperature, as at H.

The right or wrong of these ultimate conclusions of the physicist is of little importance to the engineer; but a correct understanding of the familiar thermal phenomena upon which they are based is so. It is therefore worth while to state how the doctrine of degradation—as commonly taught, if not as Lord Kelvin stated it—is inconsistent with the natural facts.

This doctrine, stated briefly, says that the hot half of the universe, consisting chiefly of the suns, is steadily becoming colder, by radiating its heat to the colder half, which latter is steadily becoming warmer by absorbing it. Therefore all temperature-differences, and with them all availability of heat for work, must ultimately die out. The fallacy here is twofold, and both its aspects are obvious to any technical graduate.

In the first place, which is the cold half of the universe? What is the average temperature of the material world? Certainly far above any temperatures familiar upon the earth's surface. The mean temperature of the earth itself, considering its interior, must be above 2,000° F. Yet it is a cold planet. The planet Jupiter, exceeding all the other planets together in mass, possesses a low red heat even at its surface. The sun, of enormous mass, several hundred times that of all the planets together, possesses a mean temperature certainly upwards of 10,000° F. Unquestionably the mean temperature of all the solid matter of the universe—using the term solid here to signify all mass-portions which can be distinguished as separate units, like the stars, etc.—is a white heat. It is only the minute fragments, scattered occasionally throughout space, remote from the incandescent centers of congregated mass, or suns, which are ever "solid" in the sense which we use in engineering.

Professor Poynting has written beautifully concerning the temperature-equilibrium of matter, showing how the external temperature of any celestial body is the result of a balance between absorption and radiation. He shows that it is only as we inspect the smaller and smaller bodies, more and more remote from the incandescent centers, that the lower temperatures are found. The temperature of solid bodies is determined almost solely by radiation—quite as it is with the articles in a room containing a fire. Those nearest the fire are the warmest, and those most remote are the coolest. As heat radiates from the incandescent suns of the heavens it penetrates regions which become colder and colder as it goes. Sooner or later it must all be intercepted and absorbed. But there is no lower limit of temperature beyond which radiation will not extend, provided it chances to escape absorption long enough. It is only the mean distance of separation between solid bodies in interstellar space which determines the mean lower limit of temperature to which radiation falls, before it experiences arrest and absorption.

Temperature, then, is not a function of *time,* except in a minor fashion, but primarily of *separation.* It is only as mass is separated into remote fragments that it can become cold, hard and solid. Time, it is true, introduces a lag into the adjustment, so that departing bodies are always hotter than the temperature of equilibrium, while approaching ones are colder; but this is merely incidental.

The earth, then, belongs decidedly to the cold half of the universe. If so, we should have observed here, according to the degradation-theory, a steady accumulation, rather than degradation, of temperature. Yet it is the obvious opposite of this which forms the main support for the degradation-hypothesis!

In the second place, the doctrine of degradation implies the existence, somewhere in the universe, of an immense mass of matter which has somehow become cold enough, and is large enough, to be capable of absorbing all the radiation from the millions of suns already known to human astronomy—not to mention the fact that we are discovering more suns every day, almost as fast as we can count and catalog them. The amount of matter requisite to do this must be enormous—far greater than the mass of all the suns together—and it must be extremely cold. For there is a rigid lower limit to temperature-range, the

absolute zero, so that matter may be warmed *up* by only a thousand degrees or so before it loses its solidity; but above there is an unlimited range of temperature through which radiating matter may cool itself *down*. Any engineer who has handled steam-condensers will appreciate the task of finding a proper cooling-medium for absorbing all the sun-heat of the heavens, without danger of overheating the medium.

There is ample interstellar space to contain all this matter—though there might be an awkward question why it did not interfere more obviously with light-transmission through space. But the unanswerable question is: *How did this matter ever get so cold?* It could not have done it by expansive labority; for that is a process confined to the gaseous substances, and could never reduce a substance to a solid, nor even make any approach thereto. Labority tends to result in a low-temperature rarefied gas, and might account for the formation of cold attenuated nebulae, but never of solid planets.

As for cooling by radiation, or thermogy, it is inconceivable that any body could ever so cool itself in that way as to turn around again and, in the same locality, proceed to absorb heat; and that at enormous rates, in enormous quantities. It is quite imaginable that a body should so cool by radiation that its rate of radiation should become *almost* zero. The cold bodies of remote interstellar space are in this condition. But it is inconceivable that its rate should ever squarely reach zero, and still less that it might ever develop a deficit of temperature, against its own radiative tendencies.

The fact is that radiation is *everywhere* cooling matter. Thermal energy never tends in any other direction than downward in temperature. *It is the very universality of radiation and temperature-drop in heat which denies the doctrine of degradation.*

Yet, in spite of this universal tendency of heat down-temperature and the resultant inconceivably vast flood of radiation pouring forth from every sun into the furthermost crevices of space, the mean temperature of the universe remains constant. Temperature is being recovered as fast as it is lost. Thermally, it never returns; but mechanically it does. As heat, it flows only downward in temperature. But it finds chance to flow down-temperature only as it flows outwardly into space; and

in remote space it can find embodiment only in exceedingly remote, rigid and inelastic solids. But in such solids is also embodied the quintessence of *mechanical* energy. The intensity and availability which has been lost to thermal forms has been regained in mechanical form.

Ultimately these remote, cold, hard solids must end their existence in collision, resulting in gasification, expansion and incandescence. The swing of the pendulum will have been reversed. The intensity and availability of mechanical energy which they embodied will have been lost, and in its place will reappear availability for radiation and elastic work-performance.

Thus there exist always and everywhere two great thermal tendencies, which are balanced against each other in cosmic equilibrium:

First: All *heat* tends always and everywhere to fall down-temperature, either by radiation or by work-performance. Nowhere in the universe has ever been observed any cessation or reversal of this tendency. Usually, the vast flood of radiant energy pouring outward into space from all the countless millions of suns finds its chance for direct radiation into remote space. This is true of the bulk of all sun-radiation; and in this form traveling outwardly at the inconceivable rate of 186,000 miles per second, it may exist for eons. Light is as permanent a form of energy as mechanical energy or heat. The universe maintains its stock of it as permanently as it does one of the space or motion-energy of celestial bodies.*

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*Of the radiance emitted by our own sun, for instance, about one hundred millionth is arrested and degraded before the confines of its own system are reached and passed. Beyond those confines, apparently, is nothing to arrest it until the next solar system is reached. When that occurs, assuming the similarity of all solar systems to our own, wherein about one-fiftieth of all the surface presented is dark, about one-fiftieth of the radiation arrested would be degraded, by collosion with opaque bodies. The other forty-nine fiftieths would be arrested by the central sun and reflected without degradation.

But even these proportions are divisions of what is itself but a minute fraction of the whole. Assuming that the mean inter-stellar distance is but eleven light-years, the proportion of the entire radiation arrested by each solar system would be only a decimal fraction of the whole consisting of unity at the seventeenth decimal place. In other words, it would not be until 100,000,000,000,000,000 solar systems had been met, after 11×10^{17} years had elapsed, that all of the original radiation from our sun would have ceased its continuous existence, and been either reflected

In small part, however, this great current of radiation finds itself intercepted by a planet equipped with air and water. It becomes entangled in the tasks of raising ocean-water into clouds, or rearing lofty forest-trees for making coal-beds, or driving reluctant engine-pistons before it as it goes. Imprisoned in the latent form of lifted weights or stored hydraulic reservoirs, or of chemical energy of wood, coal, oil or gas, it may tarry shortly here on earth before it proceeds. But only briefly. Sooner or later it has regained its thermal form and liberty and is off again, outward into space and downward in temperature, never to return voluntarily an inch or a degree, as heat. Beaten back up-temperature temporarily it may be, by superior force, as in our air-compressors and refrigerating-machines; but it always resists stubbornly and may be confined only temporarily. Soon it eludes us again, and is off for the frightful abysses of interstellar cold and darkness—to relieve them as it may.

Secondly: All *lack of heat* (or *solidity*) tends always and everywhere to increase in entropy, by impact or friction, or to decrease its embodiment in solidity. When the wandering radiation finally reaches its destination and is absorbed by some remote solid, the lower the temperature of its new home the greater must be the latter's separation from the great mass-center which did the radiating, and the greater must be its rigidity and inelasticity. Always and everywhere such remote hard solids tend to fall toward the greater aggregations of mass. This tendency is just as incessant as is that of heat downward in temperature and outward in space. So remote and hard are most of them, and so intense is their kinetic energy when they do fall, that they are fit only for the manufacture of incandescent suns. But some are quite near the surface of the earth, stored in the hills, and upon their way toward greater propinquity they too may stop a moment to perform work for us—driving our water-wheels, accelerating our railway-trains on down-grade, or, as in the case of the moon, cleansing our coasts with tidal flow.

But, like the heat, we can arrest these only temporarily. The

or degraded; and since, at each star, forty-nine fiftieths of this would have been reflected without degradation, it is not until fifty times the above number of solar systems should have been met, after 55×10^{18} years, that all of the original radiation would have suffered true degradation.

tendency is all in one direction, and is irresistible. The sand washed down from the hills never returns. Each year the earth is a smaller sphere than before, and a colder, harder one. Nothing will ever recover this ground until the earth's final collision with some dark star ends its history as an earth, and begins the story of a new sun and solar system of habitable planets.

Equilibrium between Interchangeable Forms of Energy. Thus, just as it has been found that *within* mechanical energy, and so also *within* heat, if heat be a "mode of motion," there exists an eternal equilibrium between spacial and kinetic energies, so exists also *between* mechanical and thermal energies an eternal equilibrium of two great opposed tendencies, or gravitations. Heat, *as heat,* tends always only in one direction: downward in temperature, turning into work on the way, if it must. Work, *as work,* tends always only in one direction: downward in velocity, propinquity and solidity, turning into heat as it goes. Temporarily and locally either tendency may overcome the other; but universally they are perfectly balanced. The mean availability of each form of energy remains eternally constant.

The Rejuvenation of Intensities of Energy. Thus, while it is an invariable law that the tendency of either heat or mechanical energy must be downward in intensity, so long as it retains its original form, yet it is an equally invariable law that, sooner or later, this downward tendency must result in transformation of the energy. When the energy takes its new form the intensity also takes a new form. Herein appears a most important fact, viz: *the degree of intensity of the new form of energy is independent of that of the old.*

Just how far to urge the accuracy of this statement the writer is uncertain. It might almost be said that, as far as the old intensity was *below* the mean energetic condition, the new intensity must be equally *above* mean intensity. Apparently this is the only true and consistent statement; but, because we lack a perfect means of comparing intensities of different forms, the writer prefers to make the statement tentatively, until he can pursue the question more at length.

This much is obvious, however, that when the energy has undergone a second transformation, back into its original form,

the intensity then takes a form which can be compared accurately with its degree before the first transformation. It is now obvious that the new intensity of the original form, rejuvenated by having undergone a double transformation, is quite independent of the original intensity. The final intensity is determined only by external conditions which, for the present, may be relegated to the convenient term *chance*.

This fact forms the basis for the statement of a general law of Gravitation of Intensities of Energy, as follows:

Energy tends ever, so long as it undergoes no transformation, to gravitate to a lower degree of intensity. This tendency ceases only with transformation. The energy of a mass-system can never regain intensity except by two methods: (1) *by a contribution of energy from some external mass-system, or* (2) *by undergoing a double energy-transformation, into some other energy-form and back again.**

In order to determine the extent to which this law applies to extensities of energy, as well as to intensities, we should have to go further into the relationship between different energy-forms than seems profitable here. To show where this question leads, it may be pointed out that the downward gravitation of *mechanical in*tensities, tending always toward collision, impact, friction and thermogy, is equivalent to a similar gravitation of *thermal ex*tensity, toward an increase in entropy, or a decrease in solidity. This fact has already been recognized in the statement that heat possessed two gravitational tendencies, one downward in temperature and the other outward in entropy, or downward in solidity. It now becomes clear that, of the two energetic tendencies possessed by each form of energy, one is identical with one of the two belonging to a strange form of energy on the one hand; the other is identical with one of the two tendencies possessed by a strange form on the other hand. To trace this interweaving of energy-forms in exact language is beyond possibility at present; but this correlation of intensities of different and contrasted energy-forms must be mentioned, as essential to the principle of universal energetic equilibrium.

*This law the writer has been teaching since 1899, or possibly 1898. In 1900 it was written into the MS. of his "Thermodynamics of Heat-engines," appearing in the first edition of that book. These dates are from memory only.

The Second Law of Thermodynamics. This general law of energetic gravitation just stated has been represented, in the published literature of thermodynamics, by the so-called "Second Law." This latter confines itself solely to the downward tendencies of temperature alone. The present aim is to say merely enough to show that the law regarding temperature-fall is but a narrow and special expression of a fundamental energetic principle, which runs through all known forms of energy.

It should also be added that the absurdity of calling this merely thermal, special and partial aspect of a great principle the "Second" fundamental law has now become apparent. The engineering world is now far too frequently occupied with energy-transformations to countenance longer the use of a special nomenclature for each different form of energy, independent of and inconsistent with all the others. We cannot tolerate one set of laws for mechanics, another for heat, a third for chemical action and a fourth for electricity, when all four of these sorts of the same energy are at work in nearly every engine-room in the country. The laws of energetics must be codified as such, covering all special forms of energy. Should convenience then dictate the use of a specialized version of one or more of these laws, in any particular field of engineering, it would be well enough. But the foundation should be broad and secure.

Fortunately for this, the order of importance of the basic laws of energetics appears to coincide with the order of their chronological appearance. The Conservation of Mass was discovered first, the Conservation of Energy second, and the Conservation of the other Factor of energy than mass—whether it be called Motion, Space, Propinquity, Temperature, Intensity or what you please—came last. This apparently reserves the place of Fourth, rather than "Second," for the Law of Gravitation of Intensities, which is distinctly a secondary, rather than a primary, principle.

The Summation of Energetic Intensities. The true energetic intensity of any mass-system is in reality the *sum* of the intensities of its several forms of energy. Its thermal intensity, for instance, may become very low by reason of its mechanical intensity becoming very high, and *vice versa*. But, according to the law of the conservation of energy, the sum must tend to remain constant. Where the chances of environment throw the bulk of this total intensity into any one form, there may appear

to be a creation or a loss of intensity; but it is only apparent, not real.

Every mass large enough to constitute a radiant center is constantly attracting to itself quantities of matter so cold and solid that their impact contributes enormous funds of heat. Even the earth is said to collect some twenty million meteorites daily. In these collisions occurs an example of a basic energetic phenomenon, the summation of intensities. The sun, for instance, may be taken as the hottest known body. Yet however hot it may be, it can never be hot enough to prevent a body which falls into it from increasing its energetic intensity. It may be so hot, it is true, and therefore so gaseous and elastic, that the falling body cannot make it perceptibly *hotter*. But it can and must increase its gaseous volume; and the intensity of spacial energy thus stored will abide potentially, without loss with time, to make good the temperature-drop which would otherwise occur as radiation proceeds. Incandescent radiation, like every other energetic process, cannot assume a too great intensity without producing conditions which limit its further increase.

It now becomes plain, too, why our heat-engines and other machines always have so poor an efficiency. It is because we are confined, on the surface of the earth, to a locality peculiar in being below the mean temperature of the universe. If we could only build our machines of nebular gases, instead of from wood and steel, and could jacket them with incandescence, we should soon cease our complaints of poor mechanical or thermodynamic efficiency. Indeed, if we were such salamanders as to be able to live where such procedure were natural, we should no longer prize temperature and motion as we do now. Instead of struggling ever to secure small supplies of super-temperature, to warm our bones and run our engines, and then struggling further to convert a fraction of this into much prized motion—only to have both heat and motion leak away promptly into dissipation—we should then seek everywhere for that rarest of all things: a chill and a bit of solid fixity. Everywhere would be heat. Everywhere would be motion. Flames, whirlwinds and hurricanes of gigantic dimensions would overwhelm us at every hand. Only rarely, and as a great prize, might we find a morsel of peace and quiet coolness, as a firm foundation for our

salamandric purposes. But in another instant that too would be melted, vaporized and swept away from our grasp into dissipation, by the universal surplus of heat and motion. We should then appreciate as facts what now our wits ought to teach us, viz: that cold is just as much a promoter of cycles, and is just as valuable to nature, as is heat; that solids and motion are inimical phenomena; and that happiness does not consist in always having our own way.

These facts we appreciate already, in a vague, empirical way, if we do not teach them. A machine built entirely of solids we know to be inefficient; so we insert fluid lubricating-oil between the impinging solids. Or we put our solids afloat, for efficient motion, and use an ocean of sea-water as a lubricant. Best of all, we already appreciate the efficiency of utilizing the gaseous atmosphere as a lubricant, between our rapidly flying air-ships and the earth.

CHAPTER XVI.

Transformations and Conservations.

When attention is turned to other forms of energy than work and heat, while questions as to their exact structure become more and more obscure and uncertain, yet their mutual identity with heat and work, in all of their fundamental characteristics, becomes more clear. While in the case of work it proved to be possible to analyse all features with exactness (except that no expression for the fund of tangential energy could be found), when heat was reached it became necessary to deal merely with general attributes, averaging for numbers of component mass-portions too great for individual treatment, and reserving elasticity of definition to cover conditions impossible of definition.

As the discussion proceeds from work and heat to the other forms of energy, the haziness of ideas as to exact structure—at least, when treated by the writer—must extend rapidly. It is surprising, indeed, that even an apparent identity may be discerned. Yet careful examination reveals considerable ground common to all the energy-forms.

First, *all the known forms of energy are mutually transformable.* Some part of any fund of any form of energy is always capable, under favorable conditions, of transformation into any other form; and in every case the Conservation of Energy holds true. Thus, if to the list of energy-forms already discussed there be added electricity, radiant energy (light), and biological (animal or vegetable) energy, instances of their mutual transformation familiar to the student can be found between each two, with one or two possible exceptions. This leaves out, among the familiar energy-forms, only sound; and the amount of energy involved in most audible phenomena is too small for perception after transformation into the other forms listed.

These instances of mutual transformation might be listed as follows:

Thermal to Mechanical:	Expansion under heat; the steam-engine.
Thermal to Chemical:	Dissociation; the lime-kiln.
Thermal to Electrical:	The electropile.
Thermal to Radiant:	Incandescence; all flames and lamps.
Thermal to Biological:	The direct effect of sun-heat upon vegetable and animal life.
Mechanical to Thermal:	Impact and friction; compression.
Mechanical to Chemical:	Detonation.
Mechanical to Electrical:	The dynamo or glass-plate machine.
Mechanical to Radiant:	(None known.)
Mechanical to Biological:	(None known—unless the stimulative effect of the slipper or the shingle be admitted to scientific dignity.)
Chemical to Mechanical:	The chemical fire-engine.
Chemical to Thermal:	Combustion.
Chemical to Electrical:	Primary and secondary batteries, discharging.
Chemical to Radiant:	Phosphorescence.
Chemical to Biological:	The consumption of animal tissue.
Electrical to Mechanical:	The electric motor.
Electrical to Thermal:	Electrical resistance.
Electrical to Chemical:	Secondary batteries undergoing charge.
Electrical to Radiant:	Crookes tubes.
Electrical to Biological:	Galvanism in medicine.
Biological to Mechanical:	Animal activity.
Biological to Thermal:	Animal heat.
Biological to Chemical:	The accumulation of fat and tissue.
Biological to Radiant:	Animal phosphorescence; the glow-worm.
Biological to Electrical:	Animal electricity; the electric eel.

This array of energy-transformations is most impressive and significant. While there are two combinations in the above list for which no instance is known to the writer, and while sound and magnetism would have to be added to the list to make it complete for the inanimate energies of the earth's surface, with celestial and sociological energies on beyond in either direction,

yet virtually it may be said that there is known to man an instance of mutual transformability, in either direction, between every two known forms of energy. Some of these phenomena are rare and obscure in occurrence, while some are of every-day familiarity. All in all, they cover an exceedingly diverse field of intricate natural action.

It is next to be noted that the identity of heat as a mode of mechanical motion originated from and rested upon, until recently, *nothing more than this mutual transformability.* When the mechanical equivalent of heat was once determined exactly, the identity of their natures was considered proven. Yet it is merely because the transformations between heat and work are so much more familiar than those between the other forms—or perhaps because they are the most familiar of those capable of exact definition, which electrical, radiant and biological energy are not—that their identity was foreshadowed so long ago and is now adopted with so little question. It was more than two centuries ago that Robert Boyle intimated the identity between heat and work. It is more than one century since Count Rumford proved the fact qualitatively, and almost quantitatively, upon a large scale. It is almost half a century since Joule proved it quantitatively and exactly. And yet, in this entire history of the progress of the idea of the identity between heat with work, not one bit of evidence has been adduced which is more conclusive than the universal mutual transformability of the two.

But a belief in mutual identity thus supported must extend as far as does mutual transformability, viz: to any and all combinations of energy-forms. Such evidence has to-day accumulated, for chemical and electrical energies, far beyond that existent for work and heat in either Boyle's or Rumford's time, if not also in Joule's. Their equivalence has been brought into the circle with the accuracy which was attained for heat only in Joule's hands. As to radiant, biological, sociological and celestial energies, no units for their quantitative measurement exist, and therefore coefficients of equivalence are of course impossible; but the instances of their mutual transformability, apparently with quantitative conservation, are multiplying daily in familiar experience.

It is also to be noted that even those coefficients of equivalence which have already been determined rest upon transforma-

tions in only one direction. Rumford's and Joule's work was all done in the production of heat from work. No fixed equivalence in the production of work from heat has ever been sought or found. The applicability of Joule's equivalent to work-performance by heat is pure assumption, checked only by indirect evidence as to the correctness of Carnot's law. The same is true of thermo-electric equivalence. We know that a definite amount of electricity will produce a certain amount of heat, but not that an equivalent amount of heat will produce its proper quota of electricity.

The question as to the identity of all these various forms of energy, as all being modes of motion and forms of separation between mass-particles, and as all being amenable to the laws of energetics, lies therefore in this state of settlement: That the evidence in favor of the identity of work, heat and chemical energy is so overwhelmingly great that no one to-day dares to deny it, or to suggest an adequate substitute hypothesis; yet that there exists, in the face of an only slightly less abundance of the same form of evidence, an astounding reluctance to admit the same identity between other forms.

Of these, electricity occupies an intermediate position. Static electricity has been positively identified as a matter of mass; but concerning kinetic electricity we are still forced to rely upon inference. Light and magnetism are still open to the most varied hypotheses.

The Universal Interchangeability of Energetic Form. Yet mass is merely the only accurate measure for *quantity* of matter. The next best definition of mass is as that which exerts force or stress, or exhibits strain under stress. Without the concept of mass the words "strain" or "stress" or "wave" or "pulse" lose all significance. It therefore seems to the writer that those who insist upon the doctrine that the ether, for instance, is not massive are just as reckless with language as were the earlier writers upon phlogiston or the degradation of energy. What a "strain or pulse in the ether" may be, if it does not signify the dislocation or acceleration of mass, the writer cannot imagine. The writers who use words to describe etheric action which imply naught but a reference to familiar mechanical (that is, massive) phenomena, and who yet simultaneously disclaim any such parallelism, cannot appear otherwise than as

tangling themselves in words—albeit as artistically as a Laocoön.

Even the late Lord Kelvin, in his 1907 paper before the British Association, assumes that the ether has no mass. Yet he speaks of a "pulse" or "disturbance" of the ether, and then assumes without argument that this disturbance involves energy. But if the ether is massless its disturbance would not necessarily, nor even probably, involve energy. The words "disturbance" or "wave" carry an energetic significance only when the disturbance is that of mass. For the only *known* thing which absorbs energy in its disturbance is mass. The ether can form no exception to this statement, for the ether is unknown.

Again, he says: "There is no difficulty in this conception of an utterly homogeneous elastic solid" (the ether). There is no difficulty in the concept of the ether as a solid; but there is as a homogeneous elastic solid. The only energetic systems man has been able to dissect, with mathematical accuracy, are the celestial systems; and in these the only instance of either elasticity or energy occurs as a function of *heterogeneity*—of a dissociation and interaction at a distance between two or more bodies which may be quite dissimilar, and each of which may be totally inelastic, but both of which are massive and are in motion. This elastic action at a distance is quite compatible with the "solidity" of the pair, by any criterion for solidity *except* homogeneity. The "homogeneous elastic solid" is much like Voltaire's famous "Holy Roman Empire," which turned out to be, upon inspection, neither holy, nor Roman, nor an empire. That is to say, we can have no concept of a truly homogeneous substance as possessing any qualities whatever; but the one especially complete lack-of-quality which a homogeneous body *must* have is absolute inelasticity and passivity.

The same looseness of thought and diction prevails in the frequent reference to an "indivisible unit" of matter. For a thousand centuries man has existed in ignorance of any portion of matter smaller than the tangible or visible. During the last century of this thousand he has confidently regarded the chemical atom as the ultimate indivisible unit—although not a bit of the evidence upholding the atomic theory indicated that the atom, however small a division of matter, was the *ultimate* subdivision. Nevertheless, during the last thousand days or so he has readily accepted the idea of the electron, a thousand times smaller than

the atom. Yet he now stands, apparently, just as firmly as ever for the idea that the electron is now really the ultimate unit of matter—although he already knows of three different sorts of electron, implying an internal configuration, and consequently a relation of parts.

The sensible man of the educational profession, and the sensible undergraduates as well, resent these childish inconsistencies. They know well that another thousand days may see the proof that the electron is itself divisible into very many still smaller mass-units, as yet undiscerned. They can see, both rationally and instinctively, that, whatever may be the last discovered smallest portion of mass, its energetic activity must imply at least some degree of elasticity, and therefore of heterogeneity of structure, held apart kinetically. They know that the one thing unknown to nature or the laboratory is homogeneity; that everything proves, upon dissection, to consist of a mere heterogeneous *relationship,* between parts which *individually possess none of the features evinced by the relationship.* The fact that many things, perhaps familiar things, yet remain undissected is no evidence whatever in negation of this idea.

For this great question of the identity between the several forms of energy no conclusive evidence can be expected. There can be brought to bear upon it, however, a second line of indirect evidence. This rests upon the identity of all these diverse energy-forms in their constitution and their characteristics of action.

The Universal Dualism of Dimension in Energetics. Early in the study of mechanical energy it was pointed out that this best known of all energy-forms is composed of two factors, intensity and extensity. These two factors combine, not as a sum, but as a product. Neither factor may vary toward zero except as the other factor varies toward infinity. Therefore, in spite of the unceasing gravitation of each factor toward its own zero, neither may travel in that direction with other than negative acceleration and increasing resistance, because it is thereby forcing the other away from its zero with positive acceleration.

These same characteristics apply to all the more obscure forms of energy, with fair completeness. Each form of energy

possesses a dual nature, consisting of two dimensions or factors, its own particular forms of intensity and extensity respectively. The gravitational tendency of each of these factors may be observed.

This duality of nature and identity between the several factors may be indicated by the following table:*

Form of Energy:	Factor of Intensity:		Factor of Extensity:	
	Name	Unit	Name	Unit
MECHANICAL:				
Potential:				
Approx.:	Distance	Feet	Force	Pounds
Exact:	Propinquity $\left(=\frac{1}{\text{distance}}\right)$	....	Mass-squared	$c\,(\text{Lbs.}\div G)^2$
Kinetic:				
Approx.:	Velocity	Feet-per-sec.	Mass	$\text{Lbs.}\div G$
Exact:	$\frac{(\text{Velocity})^2}{\text{Total mass}}$	$\frac{(\text{Feet-per-sec.})^2}{\text{Lbs.}\div G}$	$(\text{Mass})^2$	$(\text{Lbs.}\div G)^2$
ELECTRICAL:				
Potential:	Potential	Volt	Charge	Coulomb
Kinetic:	Potential	Volt	Current	Ampere
CHEMICAL:				
Potential:	....	....	Mass	Molecular wt.
THERMAL:				
Kinetic:	Temperature	Degree (abs.)	Entropy	$\frac{\text{B. t. u.}}{\text{Abs. Temp.}}$
Potential:	Disgregation	" "	Entropy	"

Although this table is not complete, the correspondence between the several forms of energy, in the possession of two factors or dimensions of energy, one of which is probably a function of motion, distance or force and the other of mass and its subdivision, is obvious. In mechanical energy the items are complete; except that we lack a unit of measurement, or even the familiar concept, of *propinquity* as the intensity-factor of potential energy; and we lack an exact expression for the tangential or latent fund of mechanical energy.

*The writer wishes to record here the statement, although it is impossible to support it here, that what study he has made of social and economic energies reveals the same duality of form running through them all. Some of the material for this appears in his "Cost of Competition." Much yet awaits development. Of the duality of factors, however, there can no longer be any doubt.

In electrical energy all the items are complete; but only in the case of static electricity has it been settled that the extensity-factor is identical with mass. The intensity of electricity classes itself with the other intensity-factors only by its forcefulness, with its gravitational tendency to fall; for electrical action always takes place away from the locality or condition of higher voltage toward that of lower voltage.

In chemical energy the intensity-factor is wanting, as yet. The only definite accomplishment in the line of identification is that of the *direction* of spontaneous chemical action as always coincident with increase in entropy. In this, chemical action is plainly in parallel with kinetic mechanical energy. But the bare existence of a duality of factors is as apparent in chemical energy as in the other forms. The mass factor is stated above as merely mass, whereas, to accord with mechanical energy, it should be the square of the mass. But here it must be remembered that chemical measurements concern themselves only with varying *numbers* of tiny mass-systems, each called a molecule, embodying energy in some particular chemical form of arrangement. Apparently the mass, energy and extent of mass-pairing of each molecule are the same; the quantity-measurements have to do only with the number of molecules. In this case the quantity-factor must vary, not as mass-squared, but as mass.

In thermal energy the two factors have already been sufficiently discussed.

With the other forms of energy the identification of the two factors is still more obscure. Still, so far as our knowledge of these other energy-forms goes, it falls in line with the general concepts which were based upon mechanical energy in the earlier chapters. Thus, we know comparatively little about the mechanism of the sound-wave in air; yet we know that these waves produce pressure, due to the arrest of moving particles of mass. In the case of the radiant energy of light, also, the pressure developed by reflection has been observed by several independent observers.

Even in the field of the most recently developed and least known of all the sciences, that of radioactivity, what little we know falls into line with these same fundamental concepts of energetic action. In radioactivity there are plainly two variables, the intensity of radiation and the mass. The extreme degree of

concentration itself, of energy within a given mass, is made more comprehensible by our earlier conclusions, drawn from mechanical energy, as to the unlimited ability of mass to embody energy.

Again, the intensity of radioactivity exhibits the most obvious downward tendency. One of the first tasks in identifying each newly discovered radioactive substance is to determine the time-rate at which the intensity drops. No such substance has been discovered in which the rate of radiation increased with time.

Again, the intensity of all forms of radioaction decreases, with time, by what is known as the half-rate law. That is to say, its intensity decreases by one-half in equal portions of time. If the rate be called unity at any instant, and the period of time be observed until that rate shall have fallen to one-half, then at the end of the second equal period of time the rate will have fallen to one-quarter, at the end of the third period to one-eighth, and so on. If these rates should be plotted upon rectangular coördinates, with time for the other axis, there would result a curve of the form of an equilateral hyperbola, asymptotic to both axes. In other words, the rate of radiation could never fall to zero, no matter how long it should continue to radiate; and, going backwards previously to the time of first observation, it may be said that the radiation must have begun within some fairly definite recent period, before which no radiation could have occurred without being infinite in its rate. In this the variation of radioactive intensity with time is quite similar to the form of variation, in hyperbolic function, of every other energetic function which has yet been examined in this series of papers.

Again, many of the manifestations of radioactivity are based apparently upon the linear motion of very small particles of mass. As the velocity of these particles approaches that of light, 186,000 miles per second, their mass apparently increases very rapidly. This is explained as due to the mass of the ether which must be displaced in their passage—just as the inertia of a ship is really that of the hull moving forward plus that of the water moving astern to make good its displacement. This explanation tacitly admits the massiveness of the ether. But even if this question be not entered, here is another energetic function—that between the apparent or effective mass of the moving particle

and the difference between its velocity and that of light—which is hyperbolic in form. For apparently, if the velocity of light could ever be attained by these particles, their mass would have become infinite; while for velocities far below that their apparent mass is very small.

Again, the radioactive substances tend to degrade, with time, into substances more stable chemically. Both lead and copper are thought to have been produced from radium in this way. This variation of chemical mass-pairing and intensity, within a chemical mass which remains unchanged in the aggregate, is an apparent parallel with the way in which thermal mass-pairing, or entropy, varies within a mechanical aggregation of mass which itself remains unchanged.

Energy-transformation. In all of these dual forms of energy, with factors varying in generally hyperbolic function asymptotic to two limiting zero-axes, the variation of each factor occurs in counterbalance against the other in stable equilibrium. Given sufficiently favorable conditions, this smooth fluctuation in stable equilibrium may cover an unlimited range. There is no known limit to the upward expansion of the intensity-factor, or the outward extension of the extensity-factor, except external conditions.

These, in every natural case, dictate a certain limit to the exaggeration of either energy-factor beyond a certain point. The hyperbolic energy-function here becomes discontinuous. The energetic equilibrium, previously stable, becomes unstable. Energy-transformation sets in. The appearance of the energy to human senses is abruptly altered. The gravitational law previously prevailing becomes invalid. It is only with especial care that the continuous force of even the laws of conservation—of mass, energy and intensity—may be discerned.

In mechanical energy these limits of stable equilibrium were fully discussed, and their results, in the form of either collision or dissociation, were identified. In thermal energy these same limits are visible, but their exact nature must be inferred rather than observed. Of these the most familiar are the temperature-limits at which occur fusion, vaporization and chemical dissociation. But these are wholly, or largely, internal in their equilibrium. Fusion is almost independent of pressure; vaporization is balanced against mechanical pressure; dissociation is balanced

against chemical pressure. In addition, thermal intensity is in equilibrium with mechanical pressure, determining whether work is to be performed by expansion or not. It is also in equilibrium, in the thermopile, with electrical intensity and resistance, determining whether heat shall be altered into electrical energy or not. In solutions, temperature is in equilibrium with the proportionate presence of dissolved and undissolved substance. In ignitable explosives temperature is in equilibrium with the rigidity or strength of the chemical structure of the atom, so that when more than a certain amount of thermal energy is introduced, per unit of mass, the instability of equilibrium appears most spectacularly, in violence of explosion. In the detonating explosives is exhibited a similar equilibrium between mechanical intensity and chemical resistance. Dynamite may be heated to any degree whatever, so that it ignites and burns; yet its chemical structure is stable (except for combustive action) in the face of this thermal intensity. But intensity of mechanical shock it cannot withstand, beyond a limited degree.

Indeed, the familiar phenomenon of ignition is one of the best illustrations of energetic equilibrium. Up to the ignition-point a combustible substance absorbs heat in the same way as ice does—with gradual change in entropy and temperature, in stable equilibrium. But as the ignition-temperature is reached the chemical equilibrium becomes unstable, with a resultant transformation even more striking than when ice melts.

But, since it has been said that all these illustrations of the more obscure energy-forms are explicable in terms of mechanical energy, the writer searched long for an instance of mechanical energy-transformation occurring, as the result of too great concentration and intensity of energy, in some more familiar mechanical fashion than celestial collisions. It was finally found, one summer's day, while watching a steamboat describe a long circle across a still, deep harbor, under momentum only. The water was glassy calm. The bow-waves formed themselves, on the side of the boat toward the center of curvature, into a wide uniform arc, which moved smoothly and noiselessly across the water's surface, narrowing concentrically as it came. The intensity of motion-energy in the water-waves was plainly being increased, by the concentric direction of the waves; yet here was no disturbing question of external conditions, as when the wind

drives a wave until it breaks, or a solid beach interferes with its progress and transforms its energy into heat.

If our ideas as to instability of equilibrium resultant from undue intensity of energy were correct, something ought to happen here soon; and it did. Finally, almost simultaneously around the extensive arc, the waves broke noisily into foam, although the water was deep and the air was still, from sheer overconcentration of energy. A large portion of their energy suddenly became heat and sound.

Stability in Energetic Transformations. Yet the instability of equilibrium visible in energy-transformation is itself limited in scope. A pendulum with the bob held vertically above the point of support is in unstable equilibrium. Released, it will transform its store of energy abruptly, with positive acceleration. Yet the result of this action is to remove the phenomenon from the field of instability, into one where the equilibrium is stable.

It is so with all energy-transformation. While energy-transformation is initiated only when the equilibrium is unstable, yet it occurs *always in the direction of recovery of stability.*

In mechanical energy it was noted that too great intensity in the form of propinquity begets collision and energy-transformation; but that the result of this is a form of energy, heat, in which occurs no collision. Too great an intensity of velocity, on the other hand, begets dissociation; but the result of dissociation is to transfer the mass-portions into such propinquity to other, larger systems that they are trapped there and can no longer dissociate.

Similarly with heat, unusual temperature begets energy-transformation in the form of work-performance; but the first result of work-performance is to lower the temperature and stop the transformation. Unusual lack of temperature begets thermogic energy-transformation; and the first result of this is to develop entropy, volume and elasticity, so that the thermogy is retarded.

The same is true of chemical energy, as familiarly visible in combustion. A combustible mixture of gases, if ignited, does not burn completely. Combustion is retarded by two things resultant from combustion: (1) temperature, causing dissociative tendencies which countervail the mutual attraction between fuel and oxygen; and (2) chemical pressure, due to the presence

of a preponderating proportion of the stable chemical products of oxidation. Both resistances to further combustion rest upon forms of thermochemical equilibrium.

Indeed, we are told that in such a mixture of gases there never exists a purity either of separation before combustion or of combination afterwards. That is to say, in every combustible mixture there exist before combustion some small portions of the products of combustion, in proportions determined by molecular equilibrium at low temperature. When the mixture is ignited most of the fuel and oxygen combine, but not all. A portion still remains in dissociation, held apart by the new condition of thermal, chemical and mechanical molecular equilibrium, determined by the new temperature and pressure prevailing. For each different temperature and pressure there is a different proportion of fuel or oxygen still uncombined. Often in the arts, as in gas-enginery, this proportion is sufficient to be of economic importance. More often it is so small as to be more of a chemical curiosity. But always it is there. Apparently, no exaggeration of condition will either get all the fuel and oxygen apart, or induce them wholly to combine.

In every phase of natural action this universal equilibrium is to be traced. The hot summer-day begets transformation of heat into electricity, breeds a thunder-storm and furnishes a most spectacular display of energy-transformation in unstable equilibrium. But the thunder-storm "cools the air" and recovers the weather's equilibrium; and no more storms occur until heat again accumulates unusually.

An electric current finds opportunity to enter a motor, finds there conditions favorable for transformation into motion—that is, an unusual intensity of field, coupled with unusual quantity of current across it. Motion ensues. But the immediate effect of the motion is to beget a counter-electromotive force, which so reduces the current that the transformation is reduced from a condition of unstable to one of stable equilibrium.

A stream of water, flowing across a meadow, washes easily the soil from the banks and carries it with it. But the washing is done in unstable equilibrium. On whichever side it occurs, the water is thrown in that direction, by centrifugal force, and exaggerates the departure from a straight line of flow. But, as these departures on either hand become extreme, the length of the

water-course becomes exaggerated thereby. The hydraulic "slope" of the stream becomes lessened, and its velocity of flow too low to carry longer an appreciable mass of suspended earth. So the stream adopts finally a series of S-shaped meanderings—in which the disposition to pick up soil is stably balanced against the disposition to drop it—as the form of water-course of permanent equilibrium.

In all departments of nature its every and most diverse aspect must be understood, first of all, as being the natural and inevitable result of preceding causes, acting always in stable equilibrium. The forms of not only earth, sea and solar system, but also those of vegetable and animal life, can be nothing else than the fruit of energetic evolution, reacting with former self and present environment in an eternal stability of equilibrium. The known forms have survived because they are those embodying stability of equilibrium.

In every field of activity known to man, in the energetics of moons, molecules and men themselves—in individual human life, in economics, in politics, in war and peace—the continued prevalence of stable equilibrium and apparent quiet begets an accumulation of intensity which periodically surpasses the critical limit and begets instability. Spectacular transformation of energy ensues. But the instability is always temporary; the transformation always occurs in the direction of recovery of stability. Everything works in the direction of its own demise and the birth of a new regime. Natural phenomena *never* progress smoothly and continuously. In nature as in human history, in molecular as in military affairs, in the celestial chariots of Phoebus and Aurora as in a modern automobile, matters get ahead by a series of explosions, followed by relaxations into lassitude.

It is the explosions, not the intervening periods of recuperation, which we perceive and by which we characterize the energy-form. To only one out of a hundred does "the French nation," for instance, mean aught or more than a Reign of Terror, a Waterloo and a Commune. The continuous daily life of the French people counts for nothing. Yet it is this continuous daily life which accumulates the energy become spectacular in the revolutions.

It is thus that we must abandon hope of securing satisfactory

names or attributes for any one form of energy. It is only transformations of energy which appeal to us. We know nothing about heat as heat. It is only as it enters or leaves its cryptic ant-hill that we see it. When it transforms itself into nervous shock in our bodies, or into volume, pressure or electric current in our thermometers, or into work in our engines, or into light in our lamps, we say: "Lo, here is heat!" But in none of these cases is it the heat itself which we perceive.

The Fundament of the Energetic Universe. It has been a constant care, in the preceding chapters, to dislodge the prevailing concept of energy as a flat-footed, static thing, resting upon an absolute zero of something as a supporting base, and rising therefrom in a simple, additive way. For this idea of energy—of all energy-forms, as well as for mechanical energy—a concept radically different, in two respects, is necessary. First, all energy-quantities vary on either side of a *mean energetic* value, which mean condition is itself unsupported. Secondly, the path of motion, in kinetics, as one of these variables, ranges thus, in eccentricity of conic-section orbit, on either side of the *parabola,* as the mean energetic path. The parabola is the fundamental orbit, the sole natural geometric base for all things, the orbit of unit eccentricity, embodying equal quantities of radial and tangential motion. The straight line has no place in energetics.

This needed metamorphosis of our ideas is so great as to appear impossible. Yet a quite similar transfiguration had to be, and was, accomplished in another science—astronomical kinematics—as much as three centuries ago. In the days when Vasco di Gama, Christopher Columbus, Magellan and Drake were opening the far seas to European commerce, and revolutionizing the world's ideas as to the nature and extent of its own civilization, what was the astronomical concept upon which rested their aids to navigation? The pre-Pythagorean or post-Ptolemaic, of a flat earth which served as a fundament for the heavens, relatively to which zero-plane of reference the sun "rose" and "set." By the time of Columbus the idea of the flat earth had given way before his own genius; but the earth still remained as the center of interstellar space. And with the mass of people the idea of the flat earth continued tenaciously. It did not disappear from all of our Protestant church-creeds until

within the last half-century. So prominent and able a man as President Kruger, of the Transvaal, still held to the "simple" faith in the flat earth, the supported skies and the moving sun, as the twentieth century dawned—although the intricacies and inconsistencies into which it leads are beyond bare statement here.

Columbus never met, in all his stormy voyages in tiny, top-heavy craft, natural obstacles to progress so great as he everywhere encountered, in public opinion, in the prevalence of these crude ideas as to the astronomical nature of the universe. For generations navigation suffered untold loss because of public bigotry in refusing to countenance true astronomy. Even a century later than Columbus the Gallilean and Copernican philosophies almost carried their advocates to the stake; and although the few then began to see, the rest followed slowly. The entire present wealth of these United States would not make good the losses to commerce and civilization which have been involved in the slow reluctance of mankind to abandon its reliance upon a rigid, tangible support for the heavenly bodies from an absolute base, in favor of a faith in intangible "action at a distance," and that too about an unsupported center, as a sufficient explanation of celestial mechanics.

Yet this early astronomy was no more crude—in comparison with the Gallilean-Copernican concept of a central sun, itself moving, unsupported, through space, with the earth and planets revolving about it—than is the "absolute zero," "up-and-down," rectilinear concept of energy, as an attribute of homogeneous, indivisible, ultimate matter, which holds sway to-day, when compared with the truth. Many of the statements now taught to our youth as fundamental principles of mechanical science are the exact reverse of the truth.

Yet the time of reform is now upon us. Some vital change looms imminent. Energy is now as important a topic as navigation was then. The great industrial and monetary interests are now linked with the use of natural energy in manufacture, and with the manufacture and sale of energy itself, as they were then with transoceanic discovery and commerce. Just as navigation was then the prime factor in gigantic transformations in human thought and political institutions, so is discovery in the field of energetics now the guiding cause in enormous recent and imminent changes in public opinion and democratic institutions. It

was steam-transportation, the cotton-gin and the telegraph which fifty years ago made of slavery—an institution which had existed beneficently to man since the dawn of history—an anachronism so inefficient and disturbing that its abolition was forced upon the nation, upon civilization, at whatever cost in men and money. The similar or greater changes in the form of our social organization which now promise to be forced upon us, as the inevitable result of the more recent discoveries of the telephone and trolley, the gas-engine and the steam-turbine, hydro-electric transmission-systems for light and power, wireless telegraphy and rural free delivery, are yet to be measured out in nature's laboratory. For their thorough comprehension and their safe guidance it is imperative that the run of practical men of affairs should possess accurate concepts of the internal energetic action and possibilities of large and intricately organized masses, whether of molecules or of men. But before we may hope to step into such a true comprehension of the energetic universe, purified from its present chaotic mixture of inconsistency with complexity, we must alter our point of view from its present post-Ptolemaic to a more Copernican position. We must get off the surface of the earth and rise above every-day human standards, before we may grasp the significance and the majesty of that every-day phenomenon: energy-transformation. Universal law holds true here, as elsewhere; but we, with our little factories and heat-engines, are not the fundament, nor even the center, of the universe.

It may be true, as says the writer on astronomy in the *Encyclopedia Britannica,* that the Copernican feat of removing the center of the celestial system from the earth to the sun, with its immediate unfolding of the complex mystery of the planetary system into rational simplicity, accomplished no perceptible advance in the science. It may be true that the future of physics lies solely "in the sixth decimal place." The writer does not believe either statement. The sort of astronomy which knows nothing outside of the sixth decimal place possibly was not advanced by Copernicus. But the sort of astronomy which could never have been revealed by sixteen decimal places, applied to the old ways—the sort of astronomy which fires men's minds with new ideals and devotions, which tears inside out old world-systems of bigoted faith and cruel superstition—this sort could never have lived without Copernicus and Gallileo.

But even the coldest and most mathematical science progresses thus. The re-definition of terms, the codification of laws and the projection of rational hypotheses are all as powerful aids to efficient observation as is the latter to the accurate growth of theory. And just at present—particularly in both the engineering and the economic fields—the empirical side is unquestionably overpulling on the whiffle-tree; we possess far more data than we have yet properly digested.

The Unity of All Energetic Action. There can be little question that the present broad trend of scientific progress is along the lines of an accumulation of complexity of detailed data, but with a simultaneous precipitation therefrom of an increasingly simple, consistent and unified set of underlying principles. This is true not only in pure science, but also in engineering and the other applied sciences—and, above all, in sociology, the most important of all sciences to human happiness. The problem has nowhere been better stated than by Sir William Ramsay, in his 1904 address before the St. Louis meeting of the International Congress of Arts and Sciences, in discussing the imminent problems of chemical science:

"I have already, in an address to the German Association at Cassel, given an outline of the grand problem which awaits solution. It can be stated shortly, then: While the factors of kinetic and gravitational energy, velocity and momentum, on the one hand, and force and distance on the other, are simply related to each other, the capacity factors of other sorts of energy—surface, in the case of surface-energy; volume, in the case of volume energy; entropy, for heat; electric capacity, when electric charges are being conveyed by means of ions; atomic weight, when chemical energy is being gained or lost—all these are simply connected with the fundamental chemical capacity, atomic weight, or mass. The periodic arrangement is an attempt to bring the two sets of capacity-factors into a simple relation to each other; and while the attempt is in so far a success, inasmuch as it is evident that some law is indicated, the divergences are such as to show that finality has not been attained. The central problem in inorganic chemistry is to answer the question, Why this incomplete concordance?"

But is it a fact, as Sir William states, that the factors of mechanical energy are so simply related? Is it not true that

other sciences are obscure chiefly because our mechanical concepts are confused, vague and often inconsistent? Is it not likely that, when we have swept our eyes clear of cobwebs in regarding our more familiar forms of energy, the more obscure ones may stand out in much better definition? At any rate, to do anything, however imperfect, toward the improvement of our scientific basis for this broader aspect of all the natural sciences, as mere departments of a single, consistent whole, is the highest aim to which human thought may now aspire.

Indeed, this is the basic object of all true education—as distinguished from mere training—to open the eyes to the invisible, to broaden the narrowness of view of ignorance. For this there is needed only an early inculcation of the *unity* of all nature. We do not hesitate to place early in the high-school course the doctrine of the Conservation of Matter, in spite of the infinite variety of form in which matter appears. We regard the doctrine of the Conservation of Energy, throughout similar diversity of form, as the core of our college-taught science. Why should not the parallel doctrines of the Conservation of Intensity, of the Duality of Energy-factors, and of the unity of all extent-factors with mass-subdivision, be taught as equally basic concepts?

To many writers, too, the assumption seems to come naturally that the different localities and scientific departments of universal action are quite independent of each other, or even discordant. The fact of unity, interdependence and identity seems to call for some rigid proof, before it can be accepted. They seem to forget that basic principles are always axiomatic. They seem to forget that a "proof" is nothing more than the dependence of a conclusion upon its premises, and not possibly of greater import than those premises. But to the writer the identity of all sorts of natural action lies in the axiomatic premises. It is their discordance which must be proven. Since the discovery of universal gravitation and the speed of transmission of light the universe has been unified over distances hopelessly beyond human comprehension, by bonds measurable, as to time, in terms of human life and action. Since the discovery of the mutual interchangeability of light with heat, motion, animal and vegetable life and inanimate electricity, since the invention of the spectroscope and the bolometer, the unity and identity of natural action in the most remote abysses of

interstellar space with the most familiar of every-day happenings here upon the surface of the earth have become, as was said, axiomatic. Their underlying principles are not merely similar; they are palpably identical. The burden which lies upon us is not that of proving that they are identical. It is, rather, to define in detail what are their differences.

Nor does this idea of unity mean that all forms of energy are but allotropic forms of one basic form, whether that be electrical or mechanical or chemical. It means that each is a different outward aspect of a single hidden inner nature, which latter we may never hope to comprehend. To the writer, mechanical is the most familiar form of energy; therefore he naturally refers all other extents of energy to mechanical pairs of mass, and all other intensities to visible space and motion. Yet he does not know what either mass, or space, or motion really is, and has no expectation or desire that any one will ever know. Similarly, to Professor J. J. Thomson, for instance, electrical is the most familiar form of energy; so he naturally refers all other extents of energy to electrical charge as a base. Yet he makes no pretence, I believe, that the true inner nature of the electrical charge will ever be known, however minutely we may dissect it further in the future. To Professor Ramsay, again, chemical energy is the natural base. Yet here again is no better hope of ultimate comprehension. Mere reduction into terms of something else is all that science may ever attempt. The unity, but not the ultimacy, of nature is the lesson of science.

"Energy," then, is a dual circular chain of links. Each "form" of energy constitutes a link in the circle. As we walk about this circle we may regard the different links lying nearest us, the chemical or thermal or mechanical as may be. From these combined impressions we judge the inward nature as best we may; just as we know an actor only after seeing him in many parts, under diverse make-ups. But none may say that any one of these make-ups is the actor himself.

In this great movement of human thought and action, due to civilization's current change of front from its earlier material-spiritual basis to its present ultra-energetic aspect, it is as natural that the engineer should forge to the front, as the first to understand and to do, as it was in earlier times for soldiers,

navigators and lawyers to be the leaders of men. But, if he is to rise to his opportunity in the new century, the engineer must be more broadly equipped. He must understand not only machines and individual men, but vast masses of men—not hundreds or thousands of them, but millions and tens of millions of them. As his first start toward equipment for his public and private duty he must grasp the great, fundamental principles of all energetic action. He must *understand,* as well as memorize, these three basic laws of all natural action, viz:

First: All energetic action, whether classed as celestial, mechanical, thermal, chemical, electrical, biological or sociological, operates under the same general principles in action. For in all these diverse forms, in so far as anything exact may be said about their structure, energy consists in the *subdivision* and *organization,* into specified relationships of motion and arrangement, of a mass of material. Yet this material, of itself, *possesses none of the characteristics peculiar to the whole.* The nature of a celestial system is not determined by the peculiarities of its planets, but by the peculiarities of their orbital relationships. That of a machine does not depend upon those of its component parts (so long as they come up to certain minimum requirements), but upon the way in which the engineer has put them together. The features of a chemical compound have nothing to do with those of its component elements. Gases can be combined to produce a solid, and solids to produce a gas, and vastly greater contrasts between raw material and result constantly appear in the chemical laboratory. Our deadliest poisons and best foods are both but carbon, hydrogen, nitrogen and oxygen, differently arranged. All are explained as being different *relationships,* within the molecule, of elementary atoms which are alike for all known chemical substances.

In the most varied physical aspects of electrical action it is the same. No one has gone further than Prof. J. J. Thomson in the reduction of all phenomena to mere variations in relationship between elementary components possessing only elementary characteristics; yet he works chiefly with electrical concepts. Similarly, the most intricate variety of vegetable forms of life is shown by botanists to be but a variety of arrangements of a substantially uniform vegetable cell, which is specialized about as much to develop root, stalk, leaf or flower in any one plant as it

is to embody the vast differences between one plant and another. And the same is true, to skip briefly to the other extreme of natural action, of the energetic action of men. One method of organization will make of an army a panic stricken mob; another constitutes it an invincible foe. One plan of organization within a factory leads to chronic bankruptcy; another to opulent profits. Anthropologists tell us that the peasants of modern France, which leads the world in science and art, are the exact copies, in cranial development, of their ancestors of eighty thousand years ago. But eighty thousand years ago ideas of political and economic organization were exceedingly crude.

The one necessary lesson for the clarifying of future progress is that each form of energy is defined, fitly for scientific discussion, only when we consider activities *between* its component units, carefully excluding all action which may occur *within* any "unit." Otherwise is confusion and no progress. The "unit" may be an electron, or a molecule, or a solar system, or a protoplasmic cell (in biological energy), or a man, as in sociological energy. Or, in the case of that international energetic action and reaction which has arisen with the steamship, the cable and the wireless, the unit of mass may be an entire nation. The true energy, existing *between* the units, may trade energy with its component units, or with external systems, it is true; but unless we exclude these sources and destinations from the discussion we are talking of two or three things at once. Confusion is inevitable.

Nowhere is this need of definition and clarity greater than in discussing the sociological energy existent *between* (not within) individual men. When we assemble a hundred metallic parts into a machine we regard the relationships *between* the parts as a special form of energy—mechanical—and as a fit subject for a special science, mechanics. We leave all metallurgical questions lying *within* each piece to other books and men. Electricity, again, we define as a relationship *between* electrons, positive and negative. But when we get a few millions of electrons cemented into a number of molecules, we call the relationship between the molecules a new and distinct form of energy—chemical. When we get a billion molecules organized into a protoplasmic cell or so, we call this relationship between the different chemicals still another form of energy—biological. When we get a million

protoplasmic cells arranged in organic form, and the organs specialized and federated into a sentient, reproductive animal, we regard the relationship *between* the cells and organs (which are all alike, yet all different) as still another distinctive form of energy—that of the human individual.

But when we get a hundred million men and women organized into special sexes, ages, trades and professions, and these federated and refederated into a modern State, we decline to admit that there arises therein a new and distinct form of life and energy—the sociological—*between* individuals. We insist upon dragging into the question at every point the human nature *within* each unit—each different, it is true, yet averaging as like as any million molecules. We decline to see that human institutions may themselves have an organic life, growth, reproduction and death—as independent of the myriad of individual lives, growths, passions and deaths within them, as the history of our solar system as a whole is independent of the internal natures of its component parts; which last are much more diverse than are different human natures. We fail to see that the written history of mankind records the growth, not of individual man—for evolutionary science declares this growth to have been completed before the dawn of history—but of this institutional organism of human *relationships*, an organism as distinct from individual man as the latter is distinct from his own component protoplasmic cells. That is why we fail, at present, of a consistent and satisfactory science of sociology: we have not yet taken it up as a department of universal energetics.

With our civilization now approaching a feverish paradox of farce and tragedy, with stupendous rates of production and transportation of the means for life rising in rivalry with stupefying rates of poverty, suicide, insanity and crime; with our cost of living rising while labor-saving aids multiply; with our system of exchange left as religiously to the care of chaotic antagonism of interests and duplication of effort as our systems of production have been subjected to the last refinement of coöperative organization—with all these phenomena becoming the characteristic ones of our world-civilization, the sociological doctors disagree, both as to diagnosis and remedy, more and more hopelessly. It is high time that the said doctors were sent back to school, and there impregnated with a vigorous concept of the

general principles of all energetic action. These apply most effectively to every other problem in a pretty wide and intricate universe. They will solve our sociological problems. They should be made the fundament of every college-course, whether aimed at pure science or at pedagogy, at engineering, medicine, law, the ministry, journalism or statecraft, as the ultimate goal.

Secondly: All energetic action consists in a swing of one of the two great energetic factors—number of correlated parts, on the one hand, or intensity of relationship, on the other—on either side of a central, or mean energetic, condition. In no case may any of the factors ever reach zero; none may ever reach infinity. And this swing occurs under the guidance and propulsion, as also against the resistance, of two great gravitational tendencies, one toward the consolidation and the other toward the disgregation of the component material.

These two gravitational tendencies are never directly opposed. Each is disposed laterally or transversely to the other. Within certain limits, each may act independently of the other. Between the two, therefore, may occur the greatest variety of lateral perturbations of the general swing from one extreme to the other. The pendulum of energetic conditions is not confined to a single plane, but is capable of the utmost variety of cyclical gyrations—never confining itself to any regular geometric path, and seemingly intricate in its motions beyond comprehension—yet always guided in an eternal stability of equilibrium. In whatever direction departure may be made, the departure itself begets the disposition to return. Whether the swing be confined within the critical limits, concerned merely in an exchange of one dimension of energy for the other, or whether it trespass beyond, begetting energy-transformation, the equilibrium continues ever stable. There is no activity in nature, inanimate or animate, which does not vary stably about a mean central condition, from which it never can be driven, by vagary of circumstance, more than a finite distance, or against less than a proportionally increasing resistance which must eventually reverse the process into a return.

Thirdly: This central or mean energetic condition, which neither requires nor is capable of any fixed support from any rigid base, but hangs in mid-space like the sun in the heavens, contains always an immeasurable amount of latent and invisible "tangential" energy, into which and out of which the perceptible

or "sensible" funds of radial energy pass in indefinite amoun There is no department of natural action which has proven deceptive to the engineer as that of latent energy. The instanc are legion. The most dramatic illustrations can be drawn, usual, from social energetic systems, wherein the politicians, not the statesmen, are continually deceived as to the energet possibilities latent within a people, and as to when they are like to burst forth. In 1776 Great Britain was nonplussed by th indomitable resistance of a few ragged American farmers. I 1795, when France at home was a howling mob, unable to enforc order or supply bread on the streets of Paris, her armies sca tered the combined forces of England, Prussia, Austria an Italy. In America again, in 1864, the South and the copperhea could not understand whence came the unending resistance c the North—though Gladstone's insight told him that the Sout with England behind it, was "fighting the law of gravitation."

To-day the same is true. The engineers are as blind as th politicians, in their failure to comprehend the enormous late possibilities for productive energetic action which lie in th armies of workmen of which they must always be the officer If only these armies be once organized for a single harmonio end, their energy, according to the laws of energetics, mu increase more rapidly than the first power, and possibly as fa as the second power, of the numbers involved. That single en —in order to develop this best rate of increase—must be neith mere volume of material output nor accentuation of dividend although to-day these are the sole aim of the engineer-admini trator. It must be, instead, the welfare of the *consumer*, fo whose support alone exists the entire economic system. Th present division of society, in public opinion, into several classe such as laborer, employer, capitalist, etc., of which the consum appears as merely one, must cease. In economic democrac none of these possesses any rights whatever *except as a co sumer*. In primitive nature the law of hunger drove the ar of toil inexorably. In intricate societies it must be the sam Yet in modern economics the law of supply and demand operat but remotely, obscurely and indirectly. Like a river unde ground, or one obstructed by dams and dikes and diverted int artificial channels, it flows and feeds. But that is all. It is n free and it does not control.

It is just as natural that a growing population should be increasingly self-supporting as it is that a locomotive or steamship should be increasingly able to haul more coal than it burns. But if we should design our locomotives for the much more spectacular purpose of maintaining a pyrotechnic display of sparks by night and a salvo of steam-fountains by day, rather than for coal-hauling, we might easily find that they could not haul coal enough even to supply their own consumption. The more such locomotives we built the poorer we should be.

Yet our present method of constructing economic systems—or rather, of tolerating unchanged those which we have inherited from an ignorant past—is quite as this. The real and sole reason for the existence of an economic system at all—the transfer of food from the soil to the mouth—has been almost totally forgotten. All now centers upon considerations of immediate profit from intermediate exchange, multiplied for the purpose. Our economic system operates primarily for the purposes of profitable antagonism, empty display and concealed gains—with huge incidental waste. The consumer is supposed to have all proper control over those activities, which his money alone hires into being, when, at the bargain-counter, he chooses between two or more parallel lines of those activities; which lines are generally as like as two peas. The fact that he alone pays all the bills, for raw material, labor, superintendence, finance, dividends, profits, and finally the cost of persuading himself to buy what he actively desires or urgently needs, and that therefore his right to control goes as far as does his dollar, has been quite forgotten.

Instead, intermediate means are confused, as guides in organization, with these sole ends—the sustenance and profit of the consumer. The costly strife for private profit at intermediate points, the purposeless paying of dividends for the purpose of enticing into reinvestment a remnant of those same dividends, under the guise of timid, though very "willin'" capital, and for the support of a pyrotechnic spectacle of luxury which forms no essential part of production and distribution, has overshadowed the sustenance of the real life of the community as the sole object of all business. Incidentally thereto has arisen, with the increasing complexity of invention and the arts, a rapidly increasing intricacy and intensity of confusion *between* all industrial organizations, which no business-man would tolerate for a moment

within any one of them which he controlled. With the commercial and technical press daily calling with greater vehemence for refinement of factory-organization, for better efficiency of result, the productive organization of the community as a whole is daily presenting a more hopeless chaos of cross-purposes, antagonism of interest and duplication of effort, resulting in a rapidly decreasing efficiency of result—the feeding of the consumer. With labor-saving and luxury-creating invention advancing at a rate never before known in history, the cost, difficulty, uncertainty and dissatisfaction of living are daily upon the increase.*

It is the organization, therefore, of all industrial enterprises

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*Should the reader be interested in following more in detail this growing inefficiency of our system of industry and exchange, he will find the same analyzed and displayed, for the half-century of American progress from 1850 to 1900, in the writer's "Cost of Competition." He will find there the proofs that, whereas in 1850 the efficiency of *organization*—quite aside from any question of efficiency of individual effort—was such that seventy per cent. of the effort exerted was transformed into useful result, while thirty per cent. was dissipated in commercial impact and friction, over questions of price and ownership which are of no interest whatever to the consumer, so, in contrast, in 1900 these figures had become almost exactly reversed. Of the effort now being expended within our commercial system less than one-third results in useful product; and that small fraction suffices to produce all which we now consume. Of this same effort actually expended fully seventy per cent. is being currently lost, in impact and friction due to sheer lack of intelligent organization *between* factory and factory.

The daily progress of our times, as revealed in the weekly reviews, cannot be understood except from these facts as a basis: that with the issue of every new patent, with the landing of each new immigrant, life grows more complex. The demand for extension of organization grows more urgent. It is neither the poverty nor the criminality of the immigrant that is the trouble. Both prove, upon rigid investigation, to be imaginary. The immigrant is a raw material of the greatest potential value. But, dumped upon a land wherein economic organization is proceeding at a rate far below that required by natural law—far below all other rates of progress—its effect is that of a ton of coal dumped on a furnace-fire needing a hundred-weight. Valuable as are both invention and immigration, it is their combination which is now forcing the country into economic instability. We are much further from assimilating properly, by industrial combination, the current influx of invention than we are from that of immigration. Beneficial as is the industrial combination of tangible properties into greater unity—for it is the major source of our wonderful economic progress in the recent past—it is now proceeding at a rate far below that requisite for the control of social intensities below the critical point, for the preservation of stability of equilibrium and for the prevention of explosive energy-transformations as far-reaching and destructive as those involved in the abolition of slavery. The only solution lies in the far more rapid unification of all industrial properties—at a rate nearly proportional to the square of the population.

into a single whole, along exactly the same lines as those now enforced by all business-men in the organization of individual men within these enterprises, which alone can develop within the community its quantity-factor of social energy into commensuracy with its growing needs, which alone can expand its productive capacity more than proportionally to the first power of its population. It is the engineers of the community, more than any other one class, who must perform this task. It is they, above all others, who are equipped for understanding what they are about as they do this thing.

If this task be not undertaken the critical limits of accumulated intensity will soon be passed. Intelligence will then have become impotent. Forces will have been released which must then have their brutal sway uncontrollably, until stability be regained through exhaustion. Economic equilibrium, already wavering, will have become grossly unstable. Explosion must ensue. It is not that labor will strike successfully against capital. There were as many chances for that three centuries ago as now. It is that the hundred million consumers will strike against the absurd strife and confusion now prevailing, not only between labor and capital, but between capital and capital, leading to such universal impact and friction that inefficiency of result is growing upon us apace. They will rise and overthrow in its entirety a system which, in the twentieth century, can still find no better foundation and guide than universal antagonism of interest between man and man, between enterprise and enterprise—an antagonism to-day artificially stimulated to the last degree, in a vain endeavor to rouse it to a task for which it is inherently and inevitably impotent. No imaginable expansion of intensity of economic energy can ever meet a need for its greater extensity. Its only effect can be to exaggerate, as it delays, the vigor of the inevitable reaction.

We shall be surprised, in looking back upon this crisis when it is passed, to see how largely its incurrence has been due to the fact that the business-men, factory-superintendents and mechanical engineers of a mechanical people do not understand the true nature of mechanical and allied energies.

Conservations. Throughout all this wonderful intricacy of energy-transformation, between work, heat, chemical and electrical action, light, radioactivity, vegetation, animal life, the

activities of the body politic and economic, and the latent strength of that vast whirl of international human solidarity—the accumulation which constitutes, in the eyes of mankind, the highest aim of all these combined—throughout this whole, for all time, run the three great principles of eternal conservation, the first scientific statements of immortality. All that we can see or know as a Thing, throughout all this limitless intricacy of things, proves, upon examination, to be a mere temporary *form*. It is a form of relationship, between component portions none of which possesses the attributes or abilities of the Thing itself. These attributes and abilities are the property of the *form of relationship* only. As this form is created, either from formless dissociate dust by its congregation into organized, interacting propinquity, or from senseless solidity by its comminution and disgregation into energetic sensitiveness, these attributes and abilities come into existence. As the form of relationship changes, so do the attributes and abilities. As the form of relationship melts again into formless dissociation, or solidifies into passive stolidity, the attributes and abilities disappear.

Creation, birth, life and death are of form only. The one life of the universe continues unceasingly and unvaryingly. It is the reality alone, not the form, of the universe which is eternal. It is that, too, which is imperceptible; which possesses neither attributes, character nor individuality. It is not alone that the human senses take no cognisance of aught but mere form. They take cognisance only of *change* in form. If it were not for the birth, life and death of many millions of ether-forms before the eye each second, we should see no light. Were it not for ceaseless alteration of air-pressure upon our ear-drums we should live in blank silence. Were it not for ceaseless chemical metabolism of carbon, hydrogen, oxygen and nitrogen—themselves unchanging—in vegetable life, we should live in a desert; though sun, moon and earth still circled, we should have no seasons; winter would mean a cold bare rock and summer a hot one, equally bare. Were it not for the ceaseless birth, change and death of countless cells comprising our own bodies, so that we possess none of the flesh which we inhabited a few months ago, we should know no individual life and growth. Were it not for the ceaseless procession of new-born babies into the world, of children shooting up into the bloom of adolescence, of

active lives grown seamed and scarred and feeble from the buffets of fate, of old people laid lovingly away to rest—there could be no progress and history of the human race.

What folly, then, to speak of *inducing* progress! Progress occurs because it cannot help itself. Behind it is the energy of countless eons, non-creatable and indestructible. As well invite the earth to move more rapidly about the sun, as well invite the vine to grow more rapidly than its natural rate for soil and sun provided, as to attempt to invite or force—or quell or retard, for that matter—human progress. Not only the energy, but also the forward motion, of the race is indestructible and non-creatable. As mankind entire is but a bit of microscopic growth on the surface of a tiny mass-portion whirling in space, the last and most delicate fruit of ages of upward struggle on the part of trilobite and dinosaur, so are his energies but the latest form and conservation of the measureless energy-fund of the universe, whirled back and forth across abysmal space with inconceivable speed, but incapable of being lost or retarded, or increased or accelerated, by the slightest iota.

The fundamental principles of energetic conservation upon which these conclusions rest are these—stated in terms of mechanical energy, as the simplest and most familiar form, but interpretable in terms of any and all known energy-forms.

I. **The FIRST Law of Energetics: the Conservation of MASS.** Mass is *quantity* of matter. It exists eternally. It undergoes local and temporary aggregation or disgregation, ceaselessly; but it is never destroyed nor created.

II. **The SECOND Law of Energetics: the Conservation of ENERGY.** Energy is the space-and-motion relationship between separate *portions* of mass. It exists eternally. It undergoes local and temporary accumulation, dissipation and transformation, ceaselessly; but it is never destroyed nor created.

III. **The THIRD Law of Energetics: the Conservation of INTENSITY or AVAILABILITY of Energy.** Intensity is the degree of spacial *propinquity* and of linear *motion* between the separate mass-portions. It exists eternally. It undergoes local and temporary concentration or diffusion, ceaselessly; but it is never destroyed nor created.

IV. **The FOURTH Law of Energetics: the Cons tion of EXTENSITY of Energy.** Extensity is the ext *relationship,* or *mass-pairing,* between the interacting porti mass. It is what embodies the intensity of energy, to gi latter a habitation and a name with which to do its wor exists eternally. It undergoes local and temporary accent or disguise, through the aggregation or disgregation of ceaselessly; but its sum total is never created nor destroyed.

THE END.

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